A Survey of Verifiable Delegation of Computations

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Talk Outline

Motivation

Cloud computing, Small Devices, Large Scale Computation

Generic Results for Verifiable Computation

Protocols that work for arbitrary computations

- Interactive Proofs
- Probabilistically Checkable Proofs
- "Muggles" Proofs
- Other Arithmetizations approaches (QSP)
- Implementations (Pinocchio, Snark-for-C)

Delegation of Memory

- Homomorphic MACs
- Proofs of Retrievability
- Verifiable Keyword Search

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Computing on Demand

Cloud Computing

- Businesses buy computing power from a service provider

Advantages

- No need to provision and maintain hardware
- Pay for what you need
- Easily and quickly scalable up or down

Trust Issues

- Transfer possibly confidential data to computing service provider
- Trust computation is performed correctly without errors
- Malicious or benign

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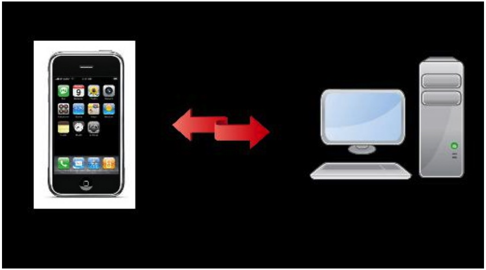
Small Devices

- Small devices outsourcing complex computing problems to larger servers
  - Photo manipulations
  - Cryptographic operations

- Same issues:
  - Confidentiality of data
  - Correctness of result



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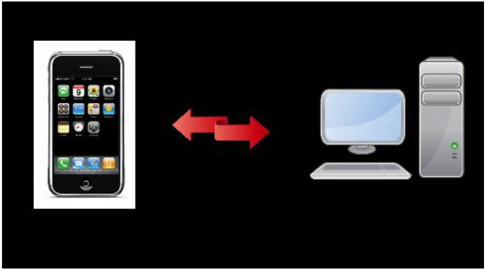

Large Scale Computations

- Network-based computations
  - SETI@Home
  - Folding@Home
- Users donate idle cycles
  - Known problem: users return fake results without performing the computation
  - Increases their ranking
- Needed a way to efficiently weed out bad results
  - Currently use redundancy

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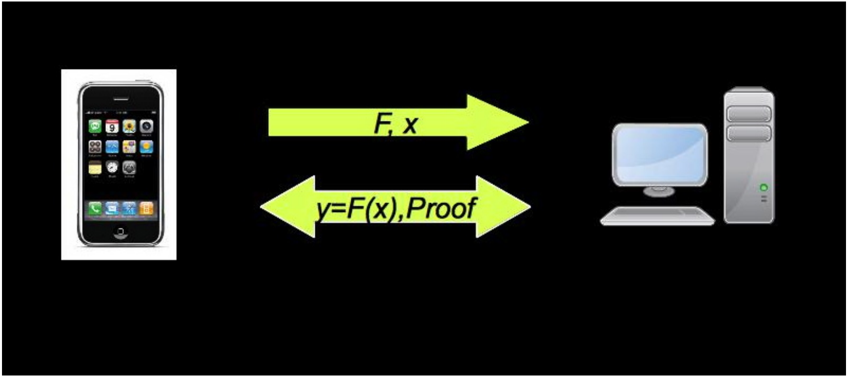
Verifiable Computation

- The client sends a function F and an input x to the server
- The server returns $y = F(x)$ and a proof Π that y is correct. Verifying Π should take less time than computing F.



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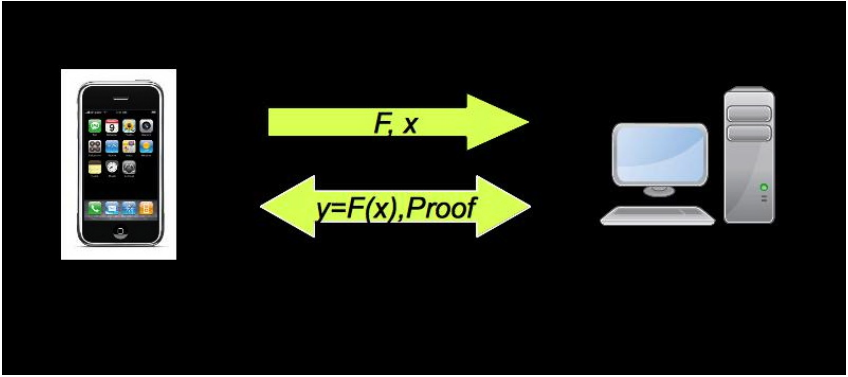
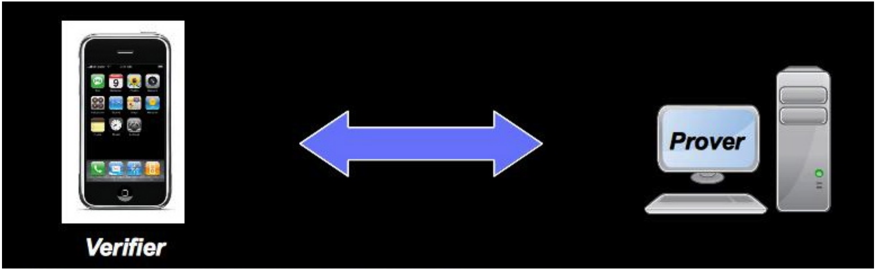


Interactive Proofs (GMR,B)

- An all powerful Prover interacts with a poly-time Verifier
  - Prover convinces Verifier of a statement she cannot decide on her own
  - Probabilist guarantee
  - All of PSPACE can be proven this way [LFKN,S]
- We want something different
  - A scaled back version of this protocols for efficient computations
  - A powerful but still efficient prover: its complexity should be as close as possible to the original computation
  - A super-efficient Verifier: ideally linear time



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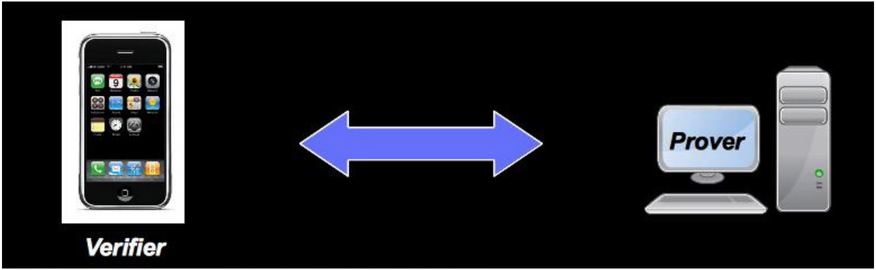
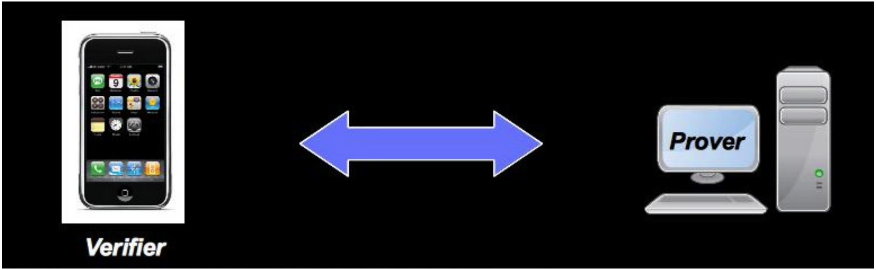


Muggles Proofs (GKR)

- Poly-time Prover interacts with a quasi-linear Verifier
  - Refines the proof that IP=PSPACE to efficient computations
- For a log-space uniform NC circuit of depth d
  - Prover runs in $poly(n)$
  - Verifier runs in $O(n + poly(d))$
  - Interactive ($O(d \cdot \log n)$ rounds)
  - Unconditional Soundness



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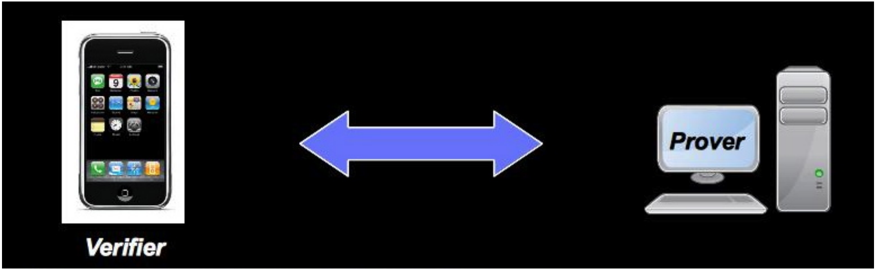

Optimizations and Implementations (CMT,T)

- Prover can be implemented in $O(S \log S)$
  - Where S is the size of the circuit computing the function
  - $O(S)$ for circuits with a regular wiring pattern
- Implementation tests show that for the regular wiring pattern case the prover is less than 10x slower than simply computing the function.
- Protocol remains highly interactive
  - Interaction can be removed via the Fiat-Shamir heuristic (random oracle model).

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Probabilistically Checkable Proofs

- The IP=PSPACE result yielded a surprising consequence: any computation can be associated with a (very long) proof which can be queried in only a constant number of locations (...AMLSS, AS, ...)
- The Prover commits to this proof using a Merkle tree and then the Verifier queries it and verifies the openings (K)
  - Note that now we have an *argument* with a computational soundness guarantee
- This protocol can also be made non-interactive using the random oracle (M) or strong extractability assumptions about the hash function used in the protocol (DL,BCCT,GLR)
- Main bottleneck: still the Prover's complexity $O(S^{1.5})$

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Arithmetization

- Turn a circuit computation into a set of polynomial equations
  - Replace each gate with a quadratic polynomial
  - Check these polynomial identities in a randomized fashion by checking them on random points
  - Use error-correcting encodings to make sure that the proof is *locally checkable* (i.e. to reduce the number of random queries to the proof)
- Can we use different arithmetizations?
  - Avoid composing long PCP proofs with compressing hash functions for a more direct way to get short proofs
  - Linear Prover complexity?
- Groth showed a different approach
  - Polynomial equations are verified in the exponent (using bilinear maps over a cyclic group)
  - A Diffie-Hellman type of assumption prevents the Prover from cheating
  - Proof is very compact without using Merkle trees
  - Drawback: quadratic prover complexity and a quadratic CRS
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Quadratic Span Programs (GGPR)

QSPs add a single quadratic step to the computation, instead of checking several quadratic equations (one for each gate)

- To check that all the wires in the circuits are correct it just requires a linear test (*span program*)
- This would be too much work for the verifier (same as the size of the circuit)
- Build two copies of the "checking" span program and test them against each other
- A QSP is defined by two sets of polynomials $V = \{v_1, \ldots, v_{n+m}\}$, $W = \{w_1, \ldots, w_{n+m}\}$ and a target polynomial t
  - We say that a QSP (V, W, t) computes a Boolean function F of n inputs if and only if
  - For all $x = (x_1, \ldots, x_n)$ s.t. $F(x) = 1$
  - t divides the product of a linear combination of subsets of V and W
    - $t|(\sum_{i=1}^{m} a_i v_i) \cdot (\sum_{i=1}^{m} b_i w_i)$
    - where $a_i = b_i = 0$ iff $x_i = 0$

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The QSP protocol

- In a preprocessing stage the Verifier publishes the values g^{s^i}, $g^{v_i(s)}$, $g^{w_i(s)}$ and $g^{t(s)}$
  - for a secret random value s.
- On input x the server finds the coefficients a_i, b_i and polynomial h such that
  - $t \cdot h = (\Sigma_{i=1}^n a_i v_i) \cdot (\Sigma_{i=1}^n b_i w_i)$
- Using the values produced by the Verifier the Prover can evaluate *in the exponent* the above equation at the point s
  - Verifier checks the equation using bilinear maps
- **Efficiency:**
  - The verifier is linear to prepare the input; constant time to verify the result
  - Prover is *quasi-linear* - the polylog overhead comes from doing polynomial division to compute h
- **Security:** requires a Diffie-Hellman type of assumption which assumes that the prover cannot divide in the exponent.

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Implementation Results

Pinocchio (PGHR)

- An end-to-end toolchain that compiles a subset of C into QSPs
- Proof size is 288 bytes regardless of what it is being computed
- Verification time is 10ms
- Prover complexity still not quite there in practice
  - About 60 times faster than previous proposals
  - Can run some lightweight computations

SNARKs-for-C (BCGTV)

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- Given a C program, they produce a circuit whose satisfiability encodes the correctness of execution of the program.
  - First the C program is compiled into machine code for `TinyRAM`
  - Then the `TinyRam` code is compiled into a circuit
- A QSP is built for this circuit
  - Use the generic concept of *Linear Interactive Proof*
  - could plug a more efficient LIP if one is found
- Slightly less efficient for the Verifier
  - Proof size 322 bytes
  - Verification time dependent on x (from 103ms to 5s for long inputs)
- A bit more efficient for the Prover
  - Were able to handle a Traveling Salesman Decider on a 200 nodes

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- Proof size is 288 bytes regardless of what it is being computed
- Verification time is 10ms
- Prover complexity still not quite there in practice
  - About 60 times faster than previous proposals
  - Can run some lightweight computations

SNARKs-for-C (BCGTV)

- Given a C program, they produce a circuit whose satisfiability encodes the correctness of execution of the program.
  - First the C program is compiled into machine code for `TinyRAM`
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- A QSP is built for this circuit
  - Use the generic concept of *Linear Interactive Proof*
  - could plug a more efficient LIP if one is found
- Slightly less efficient for the Verifier
  - Proof size 322 bytes
  - Verification time dependent on x (from 103ms to 5s for long inputs)
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  - Were able to handle a Traveling Salesman Decider on a 200-nodes

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Outsourcing Your Data

- Up to now we have considered the case of a client sending F and x to the server
  - Client's limitation is in computing time
  - Cannot compute F on its own
- What if the client's limitation is *storage*?
  - Client stores large quantity of data D with the server
  - later queries F on D and receives back $F(D)$
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Homomorphic Message Authenticators (GW)

- Client stores $D = D_1, \ldots, D_n$ and $t_i = MAC_k(D_i)$.
  - Client only stores the short key k
- Later the client submits F
  - Server returns $y = F(D)$ and t
  - Client accepts if and only if $t = MAC_k(y)$
  - Verification time may be as long as computing F – focus on storage and bandwidth
- Original idea uses homomorphic encryption
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Proofs of Retrievability (JK)

- Client stores a large file F with the server and wants to make sure that it can be retrieved without downloading the entire thing (e.g. auditing)
  - Client sends a short *challenge* c
  - Server responds with a short *answer* a
    - avoid reading the entire file to produce the answer
- A possible solution (A+,SW)
  - Encode the file F using an error correcting code $F' = Encode(F)$
  - Store each block F'_i with a *linearly homomorphic* MAC $t_i = MAC_k(F'_i)$
  - The client queries a small number (ℓ) of the blocks $F_{i_1} \ldots F_{i_\ell}$ and also sends ℓ random coefficients $\lambda_1, \ldots, \lambda_\ell$
  - The server sends back $\phi = \Sigma_j \lambda_j F_{i_j}$ and $t = \Sigma_j \lambda_j t_j$
  - The client accepts if and only if $t = MAC_k(\phi)$
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Verifiable Keyword Search (BGV)

- Client stores a large text file $F = w_1, \ldots, w_n$ with the server
  - Client sends a keyword w
  - Server responds with yes/no
  - how can we efficiently verify the answer?

- Can be handled by Merkle trees
  - $O(\log n)$ complexity (time/bandwidth)
  - Can we do better?

- Encode the file as the polynomial $F(X) = \Pi_i(X - w_i)$
  - Note that $F(w) = 0$ if and only if $w \in F$

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Verifiable Computation of Polynomials (BGV)

- Client stores a high degree polynomial $F(X) = \Sigma a_i X^i$
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- Store the MAC $t_i = ca_i + r_i$
  - r_i are computed pseudorandomly, i.e. $r_i = PRF_k(i)$
  - Client only stores random secret keys c, k
  - Let $R(X)$ be the polynomial defined by the r_i

- When the client queries the value x, the server returns
  - $y = \Sigma_i a_i x^i$ and $t = \Sigma_i t_i x^i$

- The client checks that $t = cy + R(x)$
  - Note that this requires $O(d)$ work where d is the degree of the poly
  - This can be reduced if we use *closed-form efficient* PRFs
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Keyword Search Optimizations (GPSS)

- Server has to read the entire file to answer queries
  - can we use our techniques together with some "indexing"?
- A simple "bucket-hashing" index
  - Partition words into m buckets via hashing
  - Use polynomial scheme on each bucket
  - If $m \approx n$ we get expected constant size buckets
- Allows efficient updates
  - when adding or removing a word from a bucket, re-authenticate the entire polynomial associated with it.
  - client keeps track of "state" using a "timestamp authentication scheme" (as in previous talk)
    - If using Merkle trees cost is $O(\log \ell)$ where ℓ is the number of *updates*
- Can encrypt document with additive homomorphic encryption
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- A very important problem is how to deal with updates on the memory
  - without changing the secret state of the client, the server can always ignore updates
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- Multiple clients
  - Protect information from the other clients
  - Becomes secure multiparty computation with an added constraint
    - only one party has enough resources to compute the desired functionality
  - Leverage successes in SMC.
- General VC: Explore more realistic models of computation
  - e.g. RAM
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