Shape Representation and Description

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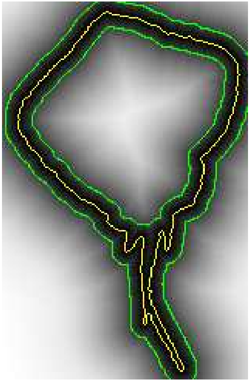
- Some feature vectors require specific distance functions to compute shape similarities independently of their orientation and size.
- The pair, feature extraction function and distance function, is called here a descriptor.

The Euclidean IFT from a contour $\mathcal{S}$ (lecture 2) creates in V multiscale contours (iso-contours) by subsequent exact dilations and erosions of $\mathcal{S}$ [1].



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- These skeletons present a highly desirable characteristic of being one-pixel-wide and connected in all scales.
- In the presence of multiple contours, a simple variant computes the skeleton by influence zones (SKIZ — a point set with equidistant pixels in at least two contours).

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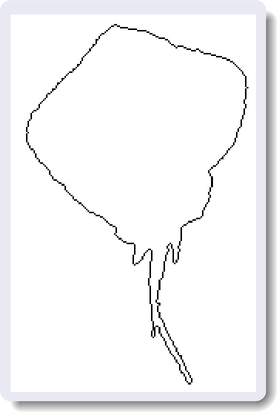

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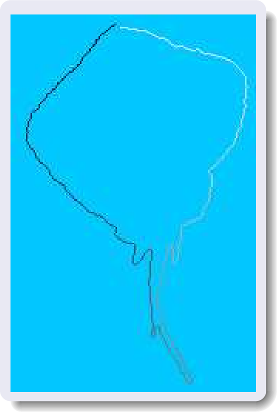

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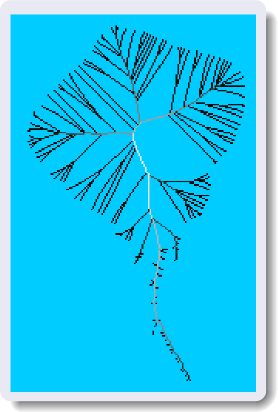

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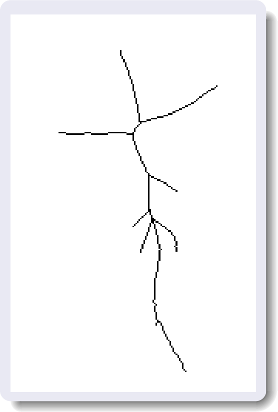

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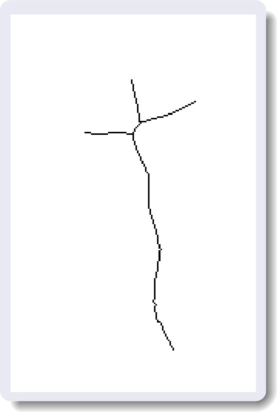

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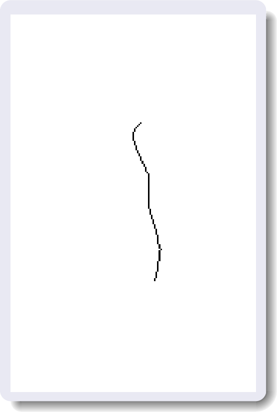

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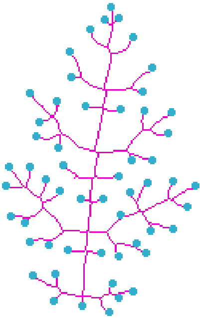

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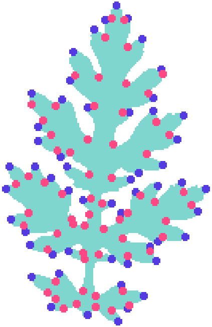

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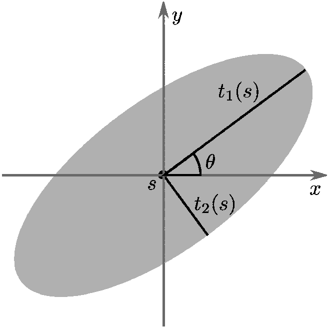
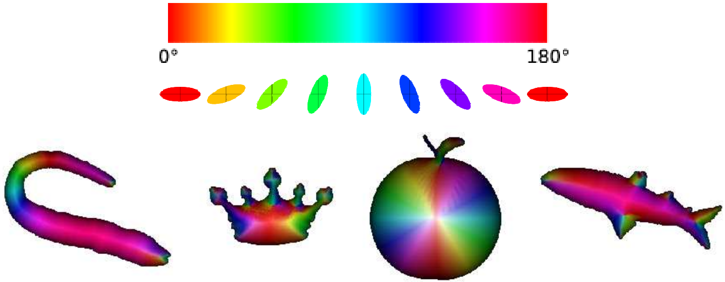
- the aperture angles of the discrete Voronoi regions in R are used to detect salience points of the skeleton,
- from salience points of the internal and external skeletons, we detect convex and concave salience points of the contour.

The Euclidean IFT can also speed up the computation of the largest ellipse (tensor scale) centered at each pixel, creating a region-based shape representation.



- *Orientation*(s) = angle between $t_1(s)$ and the horizontal axis.
- *Anisotropy*$(s) = \sqrt{1 - \frac{|t_2(s)|^2}{|t_1(s)|^2}}$.
- *Thickness*$(s) = |t_2(s)|$.

By using the HSI color space, the tensor orientation at each pixel is represented by a distinct color.



The region-based representation stores orientation and anisotropy at each pixel.

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Multiscale skeletonization and SKIZ

- Consider a binary image $\hat{I} = (D_I, I)$ with m disjoint contours $\mathcal{S}_i \subset D_I$, $i = 1, 2, \ldots, m$.

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- By circumscribing each contour in a given orientation (clockwise), a function $\lambda_p(t)$ assigns to each pixel $t \in \mathcal{S}_i$ a subsequent integer number from 1 to $|\mathcal{S}_i|$.
- Each contour pixel also receives a number $i = 1, 2, \ldots, m$ by a function $\lambda_c(t)$ to identify its contour.

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- Let $\mathcal{S} = \cup_{i=1}^{c} \mathcal{S}_i$ be the union set of all contour pixels.
- The Euclidean IFT propagates contour pixel labels in L_p and contour labels in L_c inside and outside the contours by using $\mathcal{A}_{\sqrt{2}}$ (8-neighbors) and path function f_{euc},

$$
\begin{aligned}
f_{euc}(\langle t \rangle) &= \begin{cases} 0 & \text{if } t \in \mathcal{S}, \\ +\infty & \text{otherwise}, \end{cases} \\
f_{euc}(\pi_s \cdot \langle s, t \rangle) &= \|t - R(\pi_s)\|^2.
\end{aligned}
$$

Multiscale skeletons and SKIZ

Algorithm

– Euclidean IFT with label propagation

For each $t \in D_I \backslash \mathcal{S}$*, set* $V(t) \leftarrow +\infty$ *and* $R(\pi_t) \leftarrow t$.
For each $t \in \mathcal{S}$*, do*
    Set $V(t) \leftarrow 0$, $L_p(t) \leftarrow \lambda_p(t)$*, and* $L_c(t) \leftarrow \lambda_c(t)$.
    Insert t *in* Q.
While Q *is not empty, do*
    Remove from Q *a pixel* s *such that* $V(s)$ *is minimum.*
    For each $t \in \mathcal{A}_{\sqrt{2}}(s)$ *such that* $V(t) > V(s)$*, do*
        Compute $tmp \leftarrow \|t - R(\pi_s)\|^2$.
        If $tmp < V(t)$*, then*
            If $V(t) \neq +\infty$*, remove* t *from* Q.
            Set $V(t) \leftarrow tmp$ *and* $R(\pi_t) \leftarrow R(\pi_s)$.
            Set $L_p(t) \leftarrow L_p(s)$ *and* $L_c(t) \leftarrow L_c(s)$.
            Insert t *in* Q.

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- SKIZ and one-pixel wide and connected skeletons are then obtained by thresholding $D(s)$. Higher the threshold, more simplified become the skeletons.

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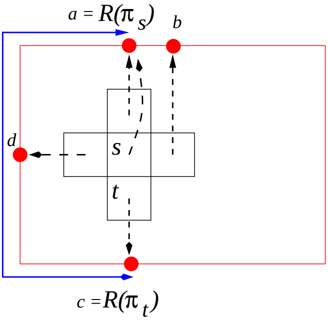
- SKIZ and one-pixel wide and connected skeletons are then obtained by thresholding $D(s)$. Higher the threshold, more simplified become the skeletons.
- Multiscale skeletons and SKIZ are computed as follows.

Multiscale skeletons and SKIZ

Each pair of contour points in $\mathcal{S}_i$ "equidistant" to a pixel $s \notin \mathcal{S}_i$ defines two segments between them. Among the shortest segments from each pair, the length of the longest one (blue line) is assigned to $D(s)$.



This condition is relaxed by computing segment lengths between root points (a, b, c, and d) related to s and its 4-neighbors.

Multiscale skeletons and SKIZ

- If $L_c(s) = L_c(t) = i$ for all $t \in \mathcal{A}_1(s)$, then

$$\begin{aligned}\Delta(s,t) &= L_p(t) - L_p(s) \\ D(s) &= \max_{\forall (s,t) \in \mathcal{A}_1} \{\min\{\Delta(s,t), |\mathcal{S}_i| - \Delta(s,t)\}\}.\end{aligned}$$

  Note that, for clockwise contour labeling, $L(a) < L(b) < L(c) < L(d)$, and the FIFO tie-breaking policy will favor the root with lowest label.

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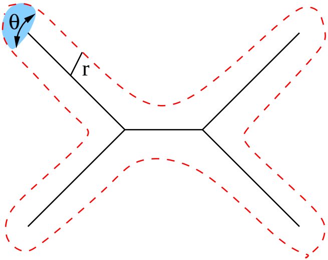
- When $L_c(s) \neq L_c(t)$ for some $t \in A_1(s)$, then the SKIZ is in between pixels s and t. Since the SKIZ is never filtered by thresholding, for $L_c(t) > L_c(s)$, $D(t) = +\infty$ and $D(s) = 0$, and for $L_c(t) < L_c(s)$, $D(s) = +\infty$ and $D(t) = 0$.

Show demo program.

Skeleton saliences

How do we compute skeleton saliences?



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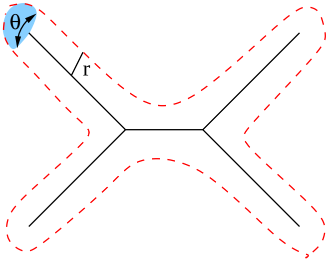

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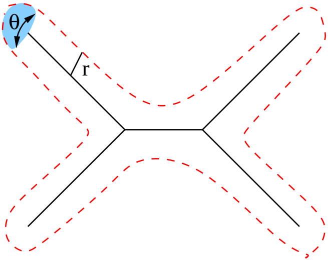

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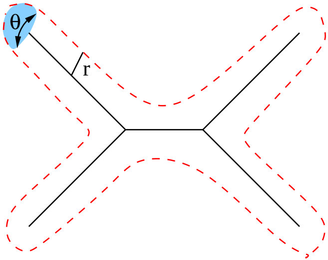

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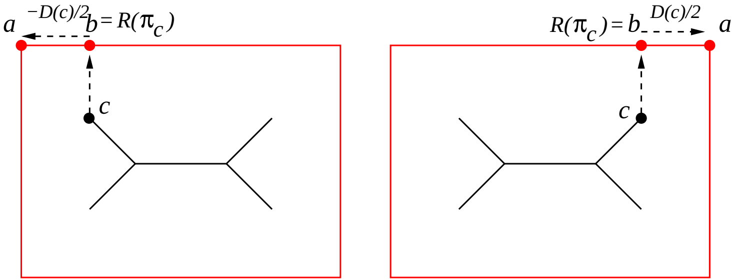
- Salience points are then obtained by thresholding θ.

How do we compute contour saliences?

For clockwise contour labeling, a contour salience a is detected from a skeleton salience c by skipping $\frac{D(c)}{2}$ pixels in either anti-clockwise or clockwise from the root $R(\pi_c)$.



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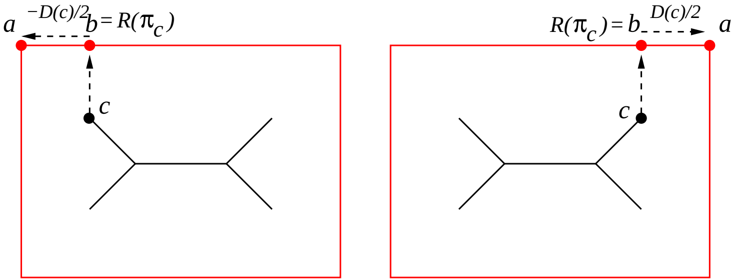


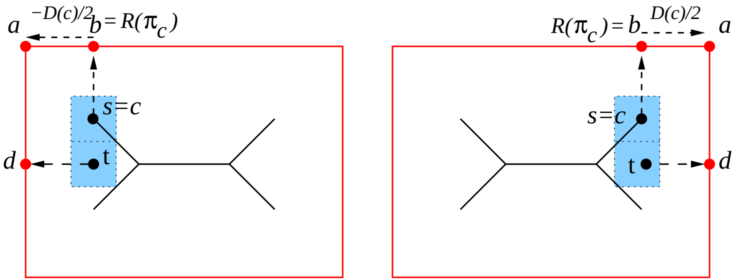
However, how do we know which orientation to go?

Contour and skeleton saliences

Let $\Delta^*(s,t) = L_p(t) - L_p(s)$ be the one which satisfies

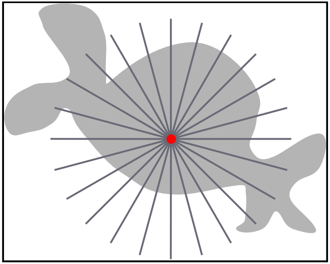
$$D(s) = \max_{\forall(s,t)\in\mathcal{A}_1} \{\min\{\Delta(s,t), |\mathcal{S}_i| - \Delta(s,t)\}\}.$$

We go anti-clockwise, when $\Delta^*(s,t) > |\mathcal{S}_i| - \Delta^*(s,t)$, and clockwise in the opposite case.



Tensor scale computation

As proposed by Saha [6], the tensor scale at s may be computed by tracing sample lines, finding edge points in each line, and fitting the largest ellipse through these points.



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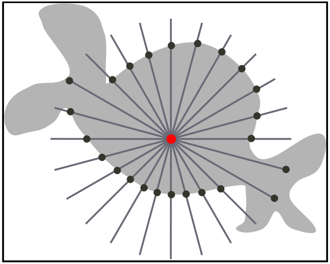

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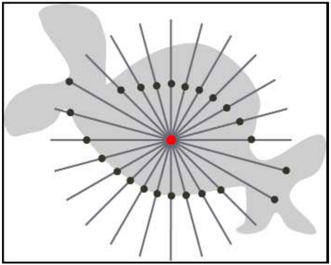

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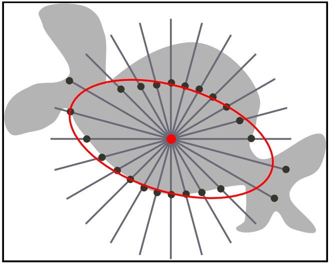

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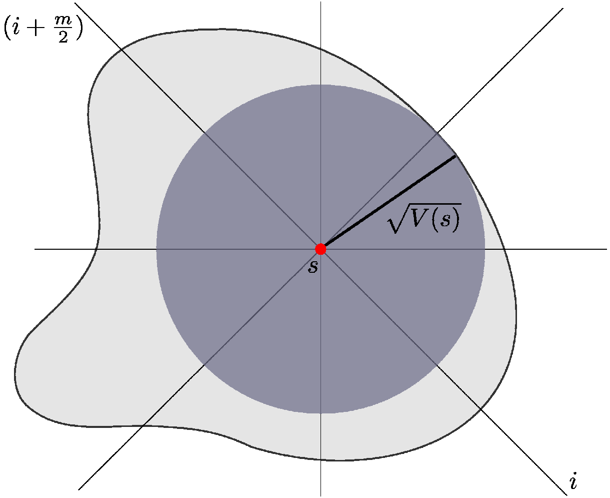

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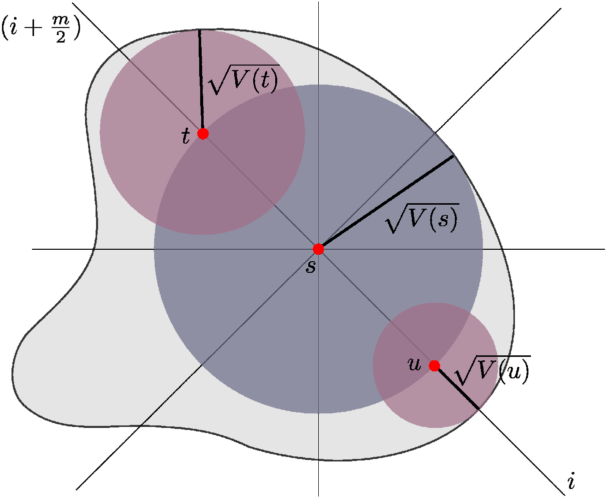

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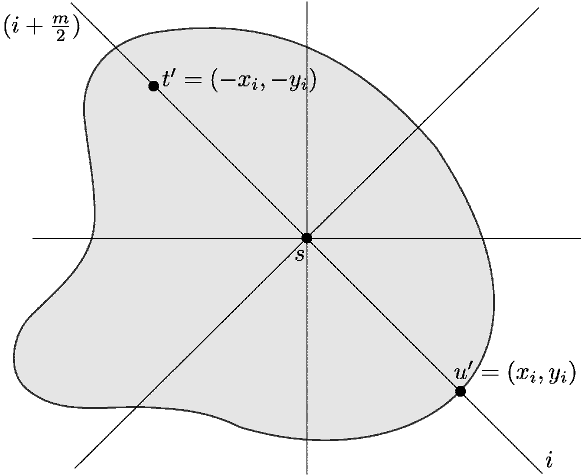

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where

- m is the number of sample lines,
- $(x_{i_\gamma}, y_{i_\gamma})$ are obtained by rotation using angle γ on the relative coordinates (x_i, y_i) of the edge points with respect $s = (x_s, y_s)$.

$$\begin{aligned} x_{i_\gamma} &= x_i \cos(\gamma) - y_i \sin(\gamma) \\ y_{i_\gamma} &= x_i \sin(\gamma) + y_i \cos(\gamma) \end{aligned}$$

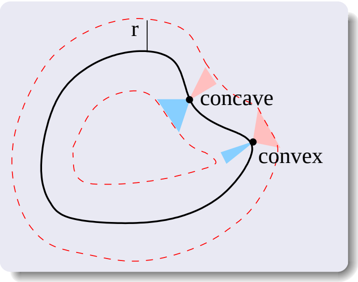
Organization of the lecture

- Muliscale skeletonization and SKIZ.
- Contour and skeleton saliences.
- Tensor scale computation.
- Shape description from these representations.
- Combining multiple descriptors.

Shape description

A descriptor is a pair (v, d), where

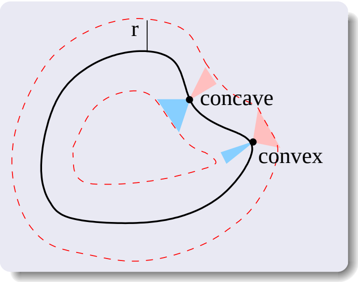
- v is a feature extraction function, which assigns a vector $\vec{s}$ to any sample s (shape, image, spel), and
- d is a distance function between samples s and t in the feature space (e.g., $d(s,t) = \|\vec{t} - \vec{s}\|$).



Feature vectors may represent

- a multiscale fractal dimension [2] computed from the distance map V.

Shape description



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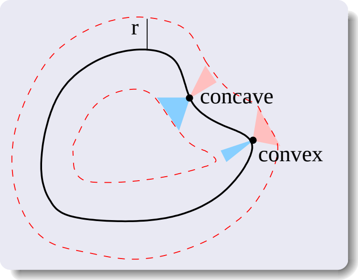

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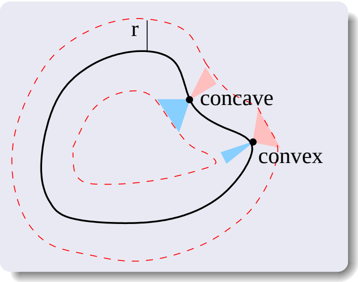

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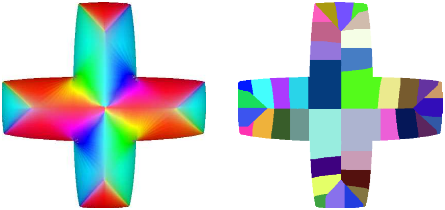
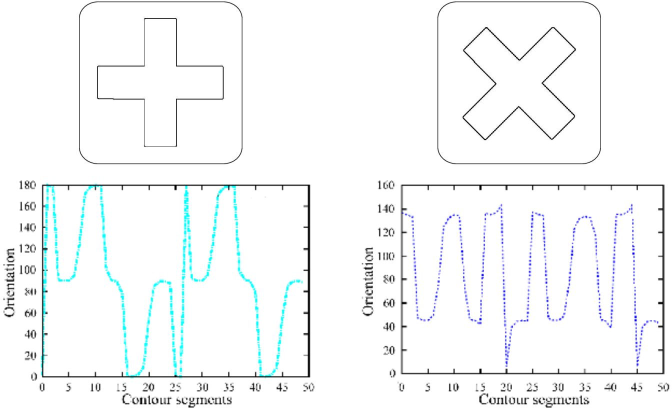
In most cases, a specific distance function is required to take into account possible shape rotation and scaling.

For example, we have divided a contour into a fixed number of segments and assigned to each segment the weighted angular mean of the orientation θ_i at each pixel s in the influence zone of that segment [3]. The anisotropy α_i of s is the weight.

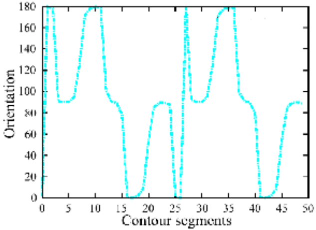
$$\bar{\theta} = arctan\left(\frac{\sum_{i=1}^{n} \alpha_i * sin(2\theta_i)}{\sum_{i=1}^{n} \alpha_i * cos(2\theta_i)}\right)$$

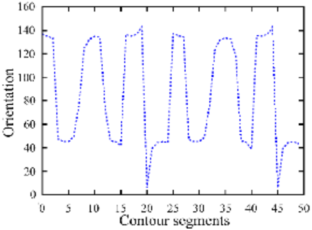


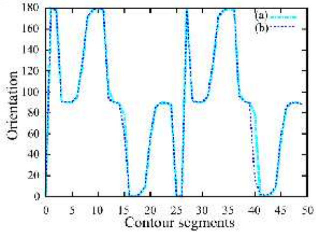
Feature vectors for a shape in different positions.



Matching between the feature vectors for distance computation.







Combining multiple descriptors

- Let $\mathbf{\Delta} = \{D_1, D_2, \ldots, D_k\}$ be a collection of descriptors $D_i = (v_i, d_i)$, $i = 1, 2, \ldots, k$, needed to handle different shape, color and texture properties.

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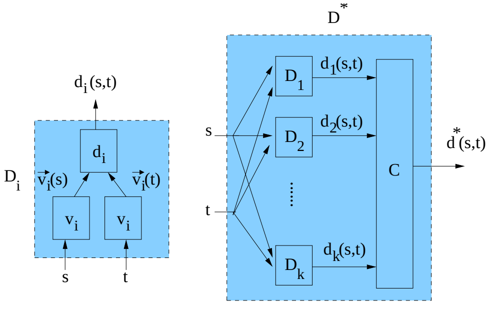
- The combination C of their distance functions is an application-dependent optimization problem which creates a composite descriptor $D^* = (\mathbf{\Delta}, C)$.



(a) (b)

We have found C by genetic programming [5].

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Conclusion

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Conclusion

- The Euclidean IFT was exploited to derive several shape representations.
- These representations involved multiscale skeletons, salience points, and tensor scale.
- Given that contour saliences are estimated from skeleton saliences, the multiscale skeletons can also obtain contour saliences in different scales.
- There are many ways to create shape descriptors from those representations and combine their distance functions into a composite descriptor.

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