Transformada Imagem-Floresta

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Introduction

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- connectivity relations: sequences of adjacent pixels (e.g., component labeling).

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- The graph nodes may be pixels, edges, regions, and the arcs will result from a given adjacency relation.
- This strategy counts with several algorithms and their proof of correctness.

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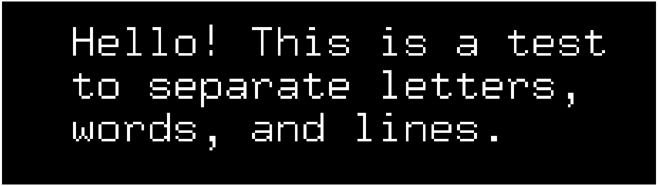
The same algorithm with distinct adjacency relations, for example, can label letters, words and lines.



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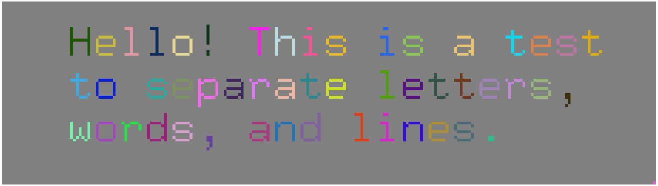

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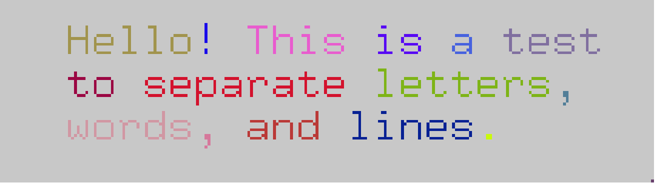

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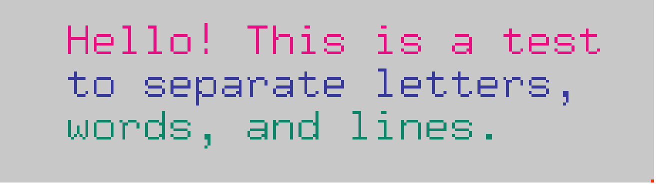

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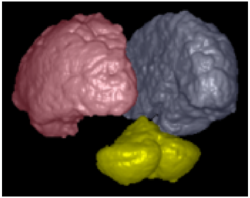

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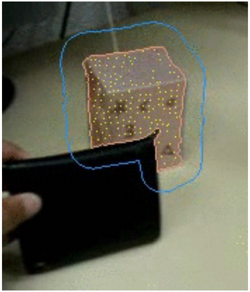

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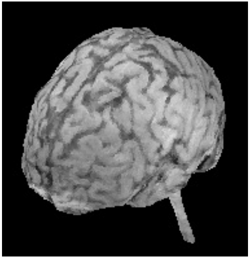

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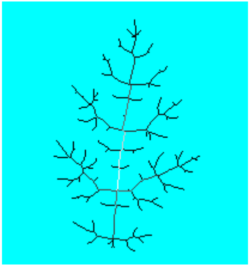

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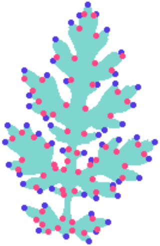

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- salience detection,

filtering, clustering, classification, etc.

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- efficient algorithms with linear execution times [2, 3, 4] in the worst case for most applications.
- a unified framework that favors a better understanding of the relation among methods [5, 6, 7] and hardware-based implementations [8], and
- effective solutions to image processing and analysis problems from the specification of a few parameters.

Organization of the course

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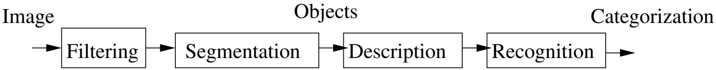

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- General algorithm, some variants and implementation issues.

General image definition

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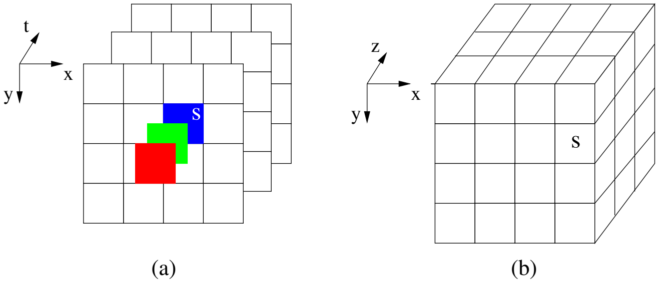
- $D_I \subset \mathcal{Z}^n$ is the image domain (a set of spels — space elements), and
- $\vec{I}(s) = (I_1(s), I_2(s), \ldots, I_m(s)) \in \mathcal{Z}^m$ is a vectorial mapping, which assigns a set of values to each $s \in D_I$.

For $m = 1$, we use $\hat{I} = (D_I, I)$.

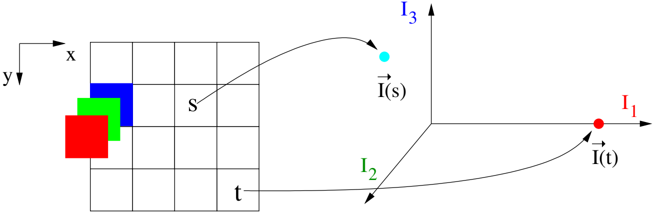
General image definition



(a) A RGB video $\hat{I}$, $n = 3$ and $m = 3$. (b) A CT image $\hat{I}$, $n = 3$ and $m = 1$.

Image domain and feature space

A spel $s \in D_I$ is a point $\vec{I}(s) \in \mathcal{Z}^m$ in the feature space.



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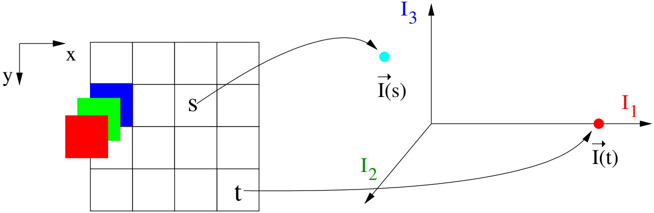

New features may result from any image transformation $\Psi(\hat{I})$, creating a real image $\hat{F} = (D_F, \vec{F})$ where $D_F = D_I$ and $\vec{F}(s) = (F_1, F_2, \ldots, F_{m'}) \in \Re^{m'}$.

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We will indicate that a spel t is adjacent to spel s either by $(s, t) \in \mathcal{A}$ or $t \in \mathcal{A}(s)$.

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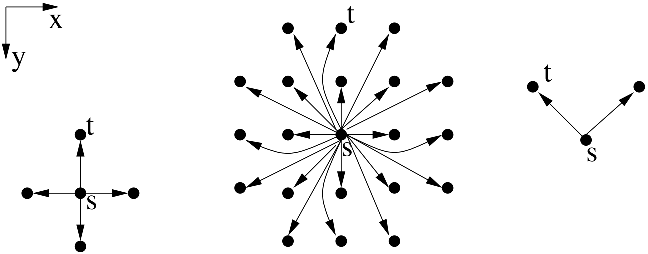

(a) Ex 1: r = 1 (b) Ex 1: r = $\sqrt{5}$ (c) Ex 2

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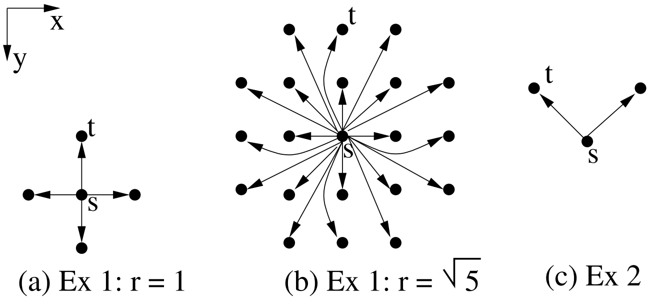

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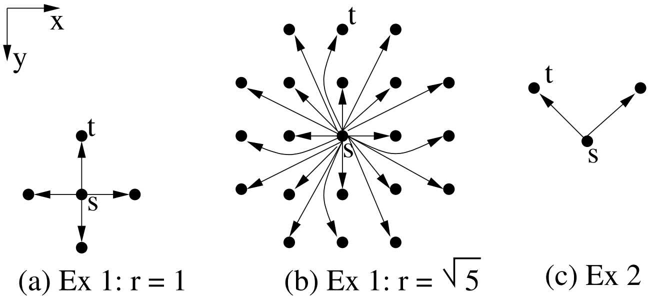

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- Example 2: $(s, t) \in \mathcal{A}$ when $t - s \in \{(-1, -1), (1, -1)\}$.
- Example 3: $(s, t) \in \mathcal{A}$ when t is a k-nearest neighbor of s in the feature space, for $k \geq 1$.

Images as graphs

Position invariant relations $\mathcal{A}$ can be represented by a vector of relative displacements

$$t - s \quad \in \quad \{(dx_1, dy_1), (dx_2, dy_2), \ldots, (dx_d, dy_d)\},$$

and fixed size $d = |\mathcal{A}(s)|\ \forall s$, leading to an implicit graph representation.

$$(x_t, y_t) \quad = \quad (x_s, y_s) + (dx_i, dy_i), i = 1, 2, \ldots, d,$$

where $t = (x_t, y_t)$ and $s = (x_s, y_s)$.

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- A convolution kernel $\mathbf{K} = (\mathcal{A}, w)$ will consist of the adjacency relation $\mathcal{A}$ and a mapping $w(t-s) \in \{w_1, w_2, \ldots, w_d\}$ of fixed arc weights.
- The reflection $\mathbf{K}'$ of $\mathbf{K}$ around its origin is simply $(\mathcal{A}', w')$, where $\mathcal{A}' = \{(-dx_1, -dy_1), (-dx_2, -dy_2), \ldots, (-dx_d, -dy_d)\}$ and $w'(s-t) \in \{w_1, w_2, \ldots, w_d\}$.

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Then, the convolution $\hat{I} * \mathbf{K}$ creates an image $\hat{F} = (D_F, F)$, by

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- Gaussian filtering with $r = 7$.
- Gaussian filtering with the above adjacency, $r = 10$ and $k = 154$.

Connectivity functions

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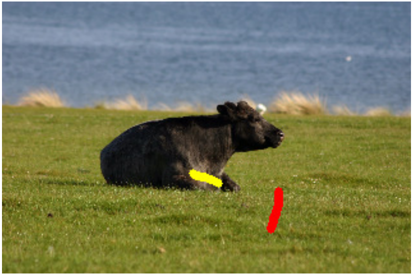
- The dual definition $f(\pi_t) \geq f(\tau_t)$ (maximum) is also valid.

Connectivity functions



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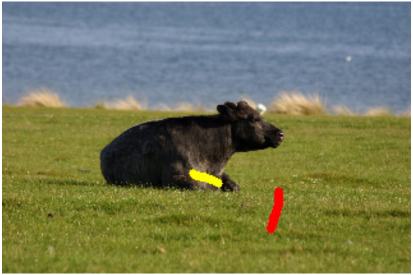

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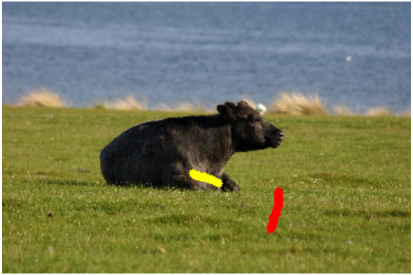

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- The seeds may compete among themselves by offering optimum paths to every spel in the image.
- The object can be defined by spels t whose optimum paths π_t have roots $R(\pi_t)$ in $\mathcal{S}_i$.

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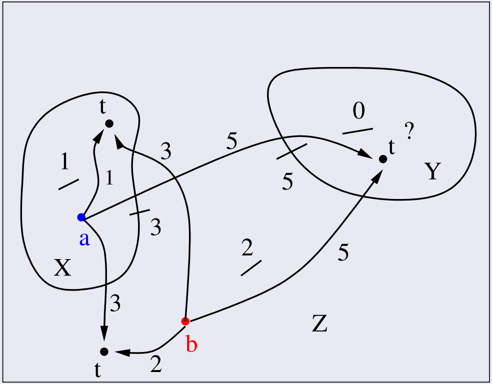
- define the value of a path to be the maximum arc weight along it, such that any path that crosses the boundary will be penalized.

The IFT computation



Seeds *a* and *b* compete between them, but *b* conquers all spels *t* in component *Y* because it will be surrounded by optimum paths rooted at *b*.

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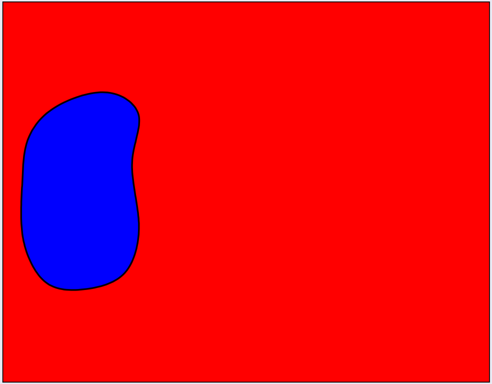

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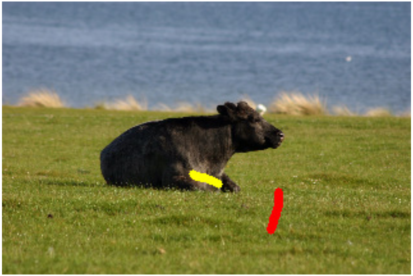

- Image with internal and external markers.

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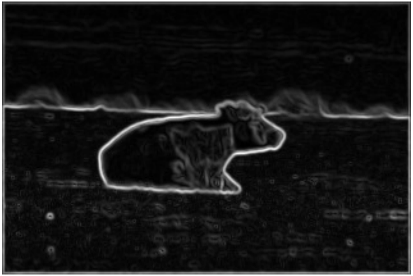
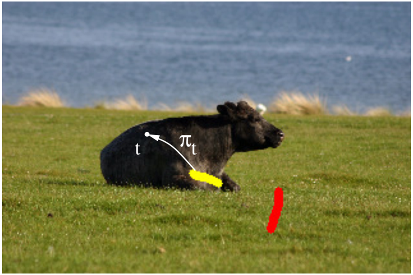


- Image with internal and external markers.
- Arc-weight image.

Connectivity functions



- Image with internal and external markers.
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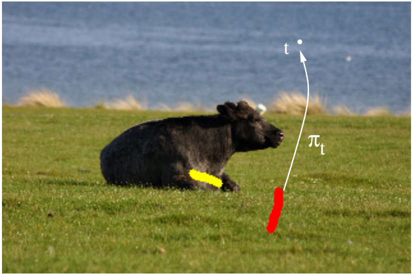
- Optimum-paths to foreground pixels.

Connectivity functions



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Connectivity functions

- Image with internal and external markers.
- Arc-weight image.
- Optimum-paths to foreground pixels.
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- Segmentation result.

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show video-iftsc.gif

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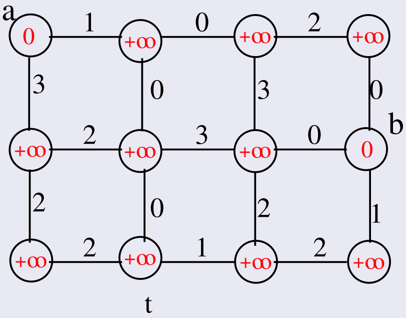
- the max-arc path function is defined as

$$\begin{aligned} f_{\max}(\langle t \rangle) &= \begin{cases} 0 & \text{if } t \in \mathcal{S} = \mathcal{S}_i \cup \mathcal{S}_e \\ +\infty & \text{otherwise} \end{cases} \\ f_{\max}(\pi_s \cdot \langle s, t \rangle) &= \max\{f_{\max}(\pi_s), w(s, t)\}. \end{aligned}$$

Connectivity functions

Consider, for example, a 4-neighborhood graph, path function $f_{\max}$ and two seeds $\mathcal{S}_i = \{a\}$ and $\mathcal{S}_e = \{b\}$.



4−neighborhood graph

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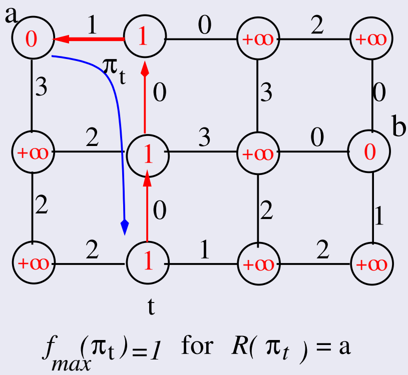

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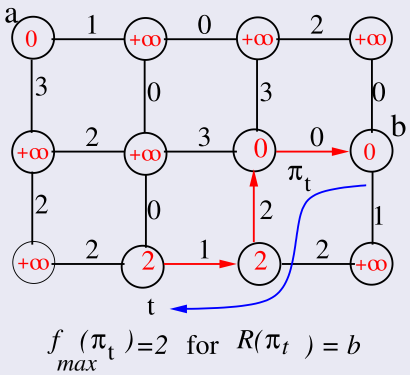

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Connectivity functions

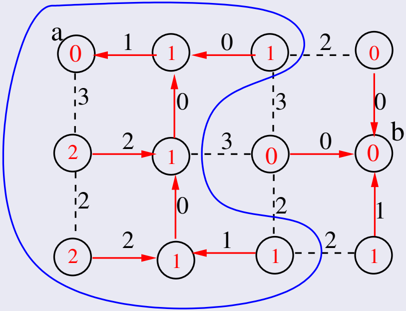
The segmentation essentially minimizes a connectivity map

$$V(t) \quad = \quad \min_{\forall \pi_t \in \Pi(\mathcal{N},\mathcal{A},t)} \{f_{\max}(\pi_t)\}$$

by considering the set $\Pi(\mathcal{N}, \mathcal{A}, t)$ of all paths with terminus t and function $f_{\max}$.

Connectivity function

The IFT algorithm solves this problem by computing an optimum-path forest P in $(\mathcal{N}, \mathcal{A})$ — a predecessor map with no cycles, containing all optimum paths from a root set $\mathcal{R}$, which in this case is $\mathcal{S} = \mathcal{S}_i \cup \mathcal{S}_e$.



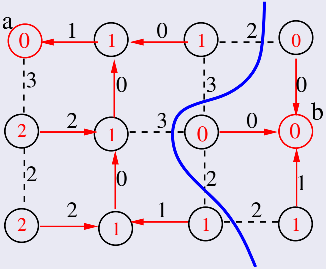
The object is defined by the optimum forest for $f_{\max}$ rooted in $\mathcal{S}_i$.

Connectivity function

An optimum-path forest for $f_{\max}$ also provides the graph cut whose minimum arc weight

$$\min_{\forall (s,t) \in \mathcal{A}, R(\pi_s)=a, R(\pi_t)=b} w(s,t)$$

is maximum, considering all possible cuts between a and b [6].



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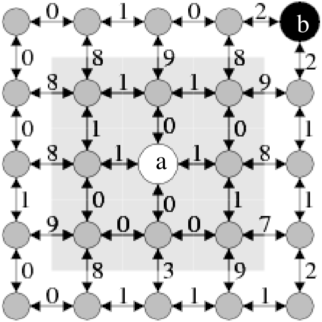
- Essentially the minima in $V_0(t)$ compete among themselves and some of them become roots in $\mathcal{R}$, being also minima in $V(t)$. show video-iftsc-computation.gif

Path propagation

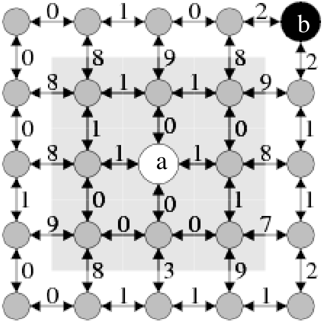
Consider the optimum path propagation for $f_{\max}$ from $\mathcal{S} = \{a, b\}$ in the 4-neighborhood graph below.

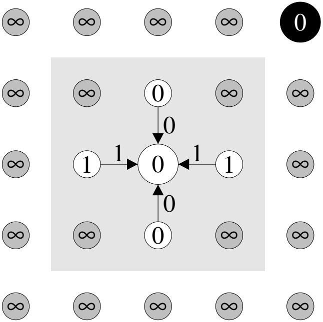


Object and background are separated by the arcs with higher weights.

Path propagation

From iteration 1 to 5, iteration 12, 20, and 25.





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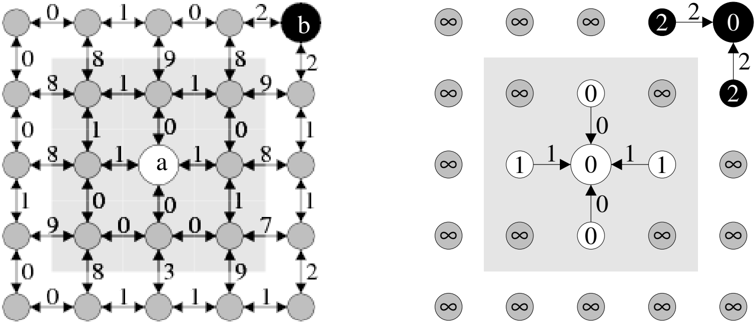

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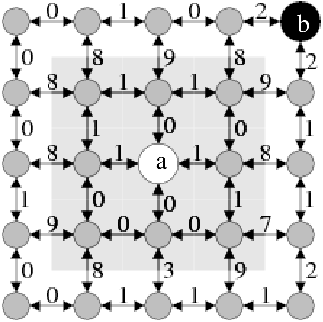

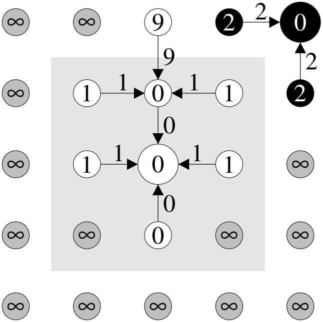

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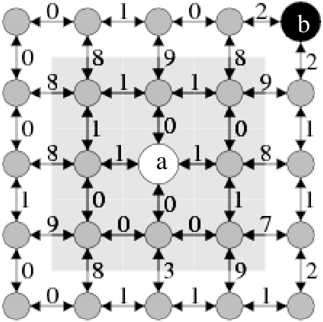

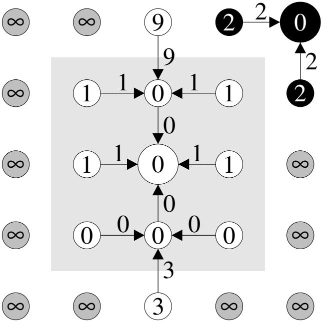

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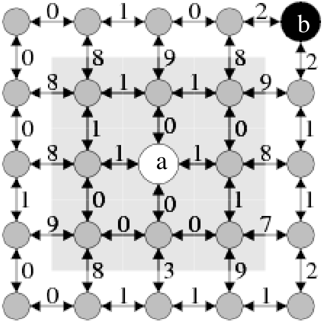

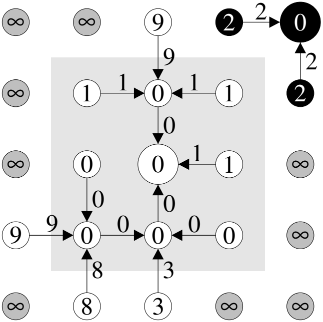

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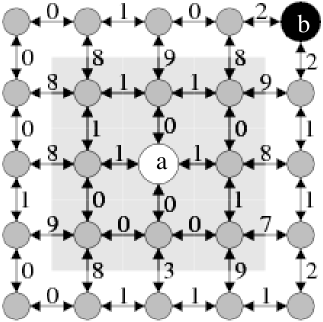

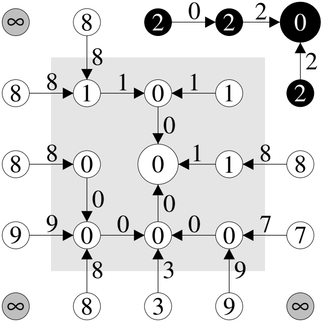

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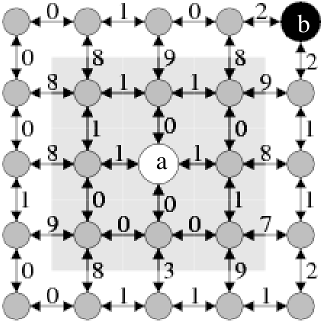

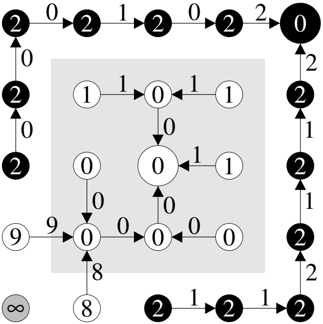

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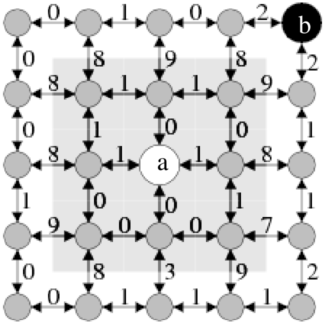

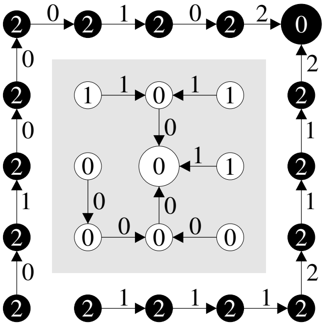

Information propagation

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- For its correctness, verify `www.math.wvu.edu/~kcies/SubmittedPapers/SS17DijkstraCharacterization.pdf`

The general IFT algorithm

Algorithm

– GENERAL IFT ALGORITHM

```
For each $t \in \mathcal{N}$, do
    Set $P(t) \leftarrow nil$, $L(t) \leftarrow t$ and $V(t) \leftarrow f(\langle t \rangle)$.
    If $V(t) \neq +\infty$, then insert $t$ in $Q$.
While $Q$ is not empty, do
    Remove from $Q$ a spel $s$ such that $V(s)$ is minimum.
    For each $t \in \mathcal{A}(s)$ such that $V(t) > V(s)$, do
        Compute $tmp \leftarrow f(\pi_s \cdot \langle s, t \rangle)$.
        If $tmp < V(t)$, then
            If $V(t) \neq +\infty$, remove $t$ from $Q$.
            Set $P(t) \leftarrow s$, $V(t) \leftarrow tmp$, $L(t) \leftarrow L(s)$.
            Insert $t$ in $Q$.
```

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- Line 7 can be simplified to $tmp \leftarrow \max\{V(s), w(s,t)\}$ in the case of $f_{\max}$.
- The dual operation $V(t) = \max_{\forall \pi_t \in \Pi(\mathcal{N},\mathcal{A},t)}\{f_{\min}(\pi_t)\}$ requires: $V(t) \neq -\infty$ in Lines 3 and 9, $V(s)$ is maximum in Line 5, $V(t) < V(s)$ in Line 6, and $tmp > V(t)$ in Line 8.

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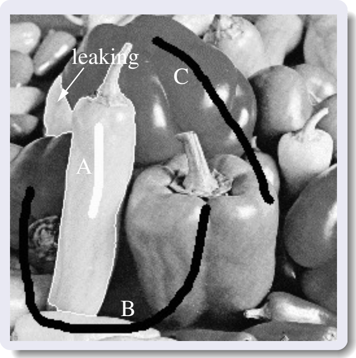
- Early termination is possible whenever s is a destination node [2] or when $V(s)$ is greater than a given threshold [9].
- Later, whenever necessary, the remaining optimum paths π_t with $V(t) \geq V(s)$ can be obtained incrementally from the nodes in Q.

Important remarks and variants

For a fixed path function f, the optimum forest can be updated in a differential way whenever we add/remove root nodes [3].



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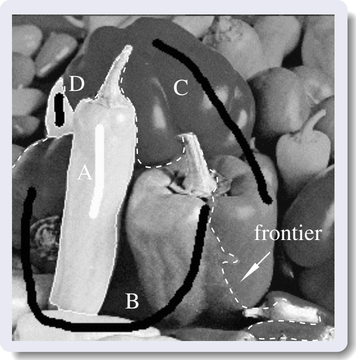

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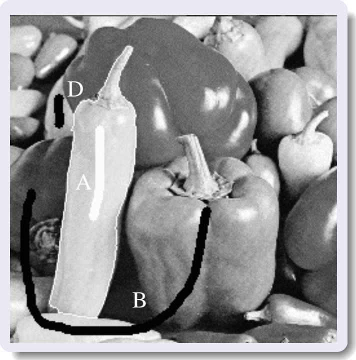

- Internal (A) and external (B and C) markers are selected, but a "leaking" occurs.
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- Result of segmentation.

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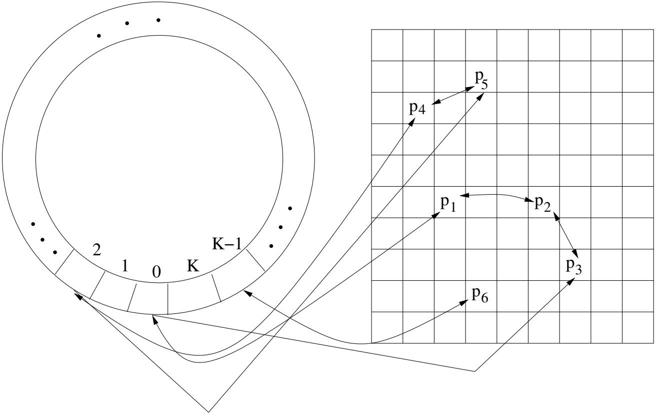
- Its running time is $O(|\mathcal{N}| \log(|\mathcal{N}|))$, when Q is a binary heap and the graph is sparse ($|\mathcal{A}| \ll |\mathcal{N}|^2$).
- Its running time is $O(|\mathcal{N}|)$, when $f(\pi_s \cdot \langle s, t \rangle) - f(\pi_s) \in [0..K]$, $K \ll |\mathcal{N}|$, are integers, the graph is sparse, and Q uses bucket sort [2].

Priority queue



Nodes t are inserted in bucket $V(t)\%(K+1)$ (left), forming $K+1$ lists (right). The property $f(\pi_s \cdot \langle s,t\rangle) - f(\pi_s) \in [0..K]$ guarantees that nodes with different values are never in a same bucket.

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- The IFT plays an important role in this scenario, given that it unifies image transformations based on optimum connectivity.
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