Reformulações e Algoritmos de Solução para o Problema de Árvore Geradora com Máximo número de Folhas

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Abilio Lucena
Nelson Maculan

IC - UNICAMP

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Outline of Presentation

Background Material

Max Leaf Spanning Tree Problem

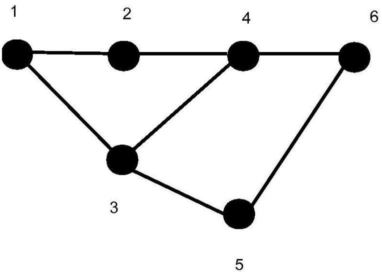
Formulation + Reformulations

Computational Results

Conclusions

Graph G

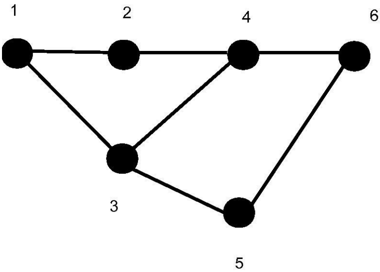
Graph $G = (V, E)$: vertex set V and edge set E.

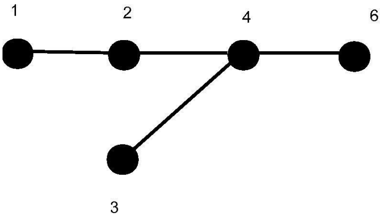


Tree $\mathcal{T}$ of G

$\mathcal{T} = (V', E')$, $V' \subseteq V$, $E' \subseteq E$

- Connected subgraph of G with $|E'| = |V'| - 1$
- Not necessarily spanning for G
- $\mathcal{T}$ may eventually consist of a single vertex of G





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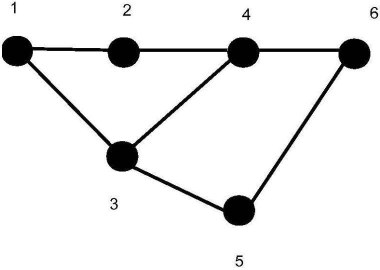

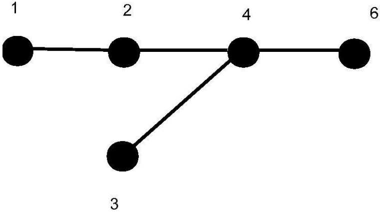

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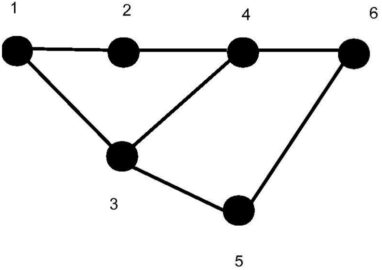

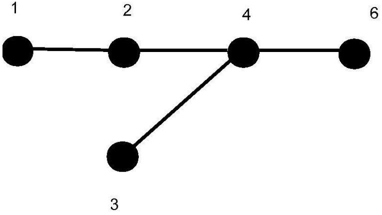

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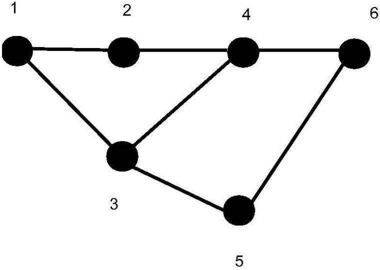

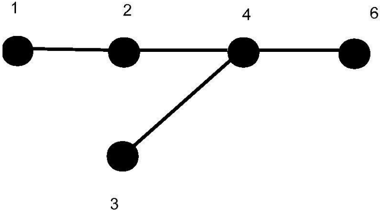

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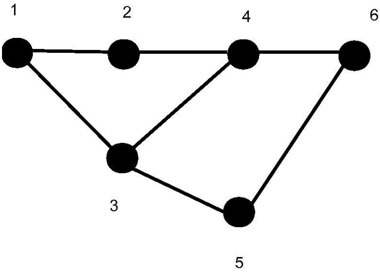
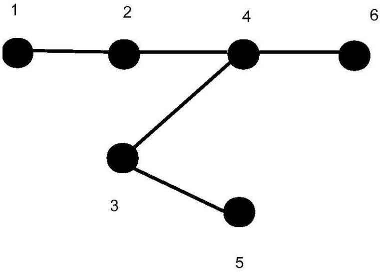
Spanning trees of G

$\mathcal{T} = (V', E')$, $V' = V$, $E' \subseteq E$: All vertices of V are involved!





Minimum Spanning Tree Problem (MSTP)

Given: $G = (V, E)$ and $\{c_e : e \in E\}$

Find: least cost spanning tree of G

Polynomial time solution algorithm (Kruskal (1956))

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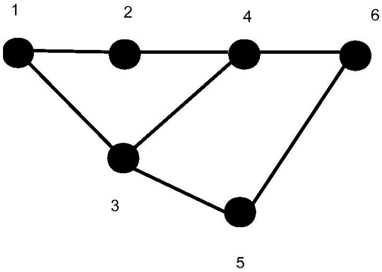

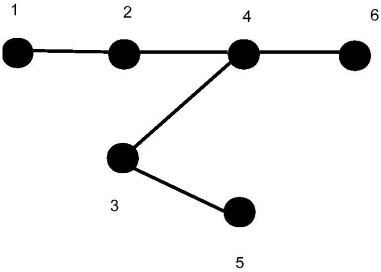

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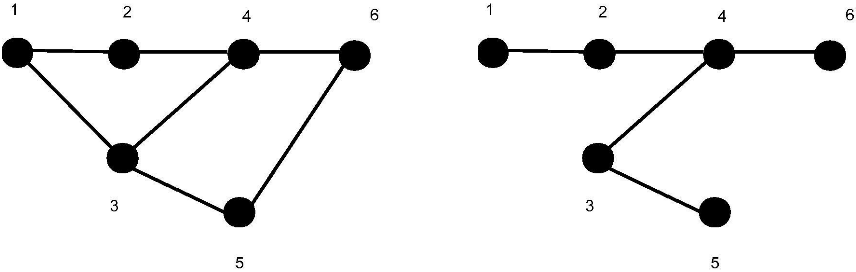

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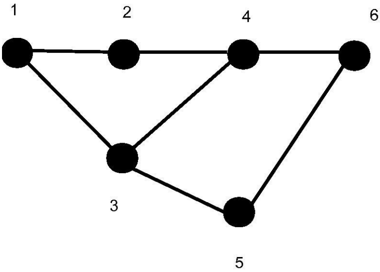

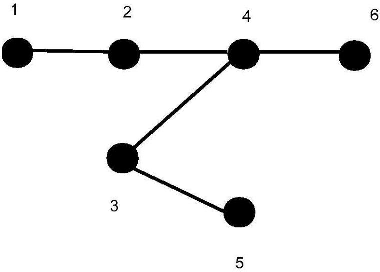

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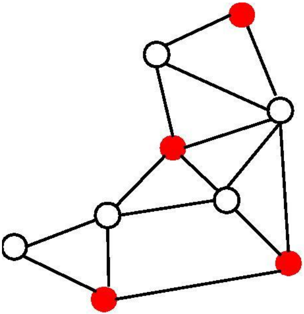
Find: least cost tree of G that spans T



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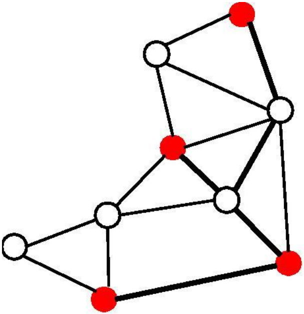

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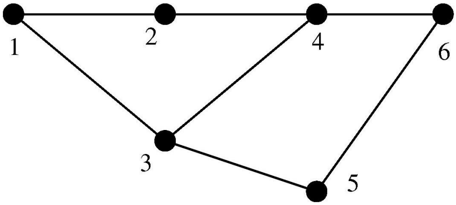
Given: a graph $G = (V, E)$

Find: spanning tree of G with the largest number of leaves



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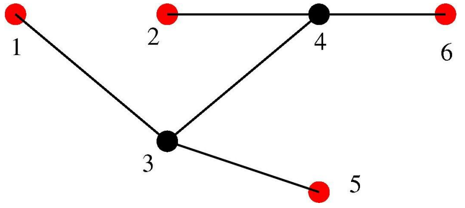

Optimal solution has 4 leaves.

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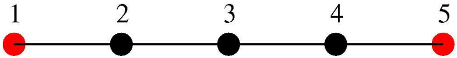

Worst case - Difficult to find optimal solution: 2 leaves.

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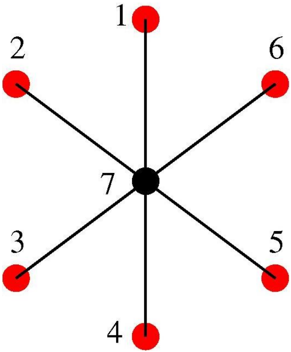


Best case - Trivial to obtain optimal solution
for complete graphs: $(|V| - 1)$ leaves

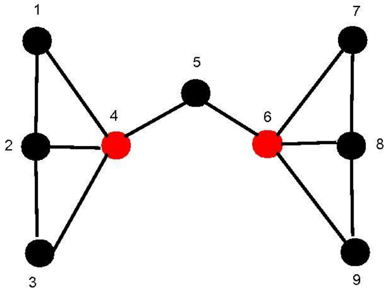
Equivalent to the problem

Dominating set of G: $C \subset V$ such that

- for every $i \in V \setminus C$: edge with an endpoint in i and the other in C.

Minimum Connected Dominating set C (MCDS)

- Implies a tree of G for non-leaf vertices of $\mathcal{T}$.
- Straightforward to generate a Max Leaf Spanning Tree of G.



Equivalent to the problem

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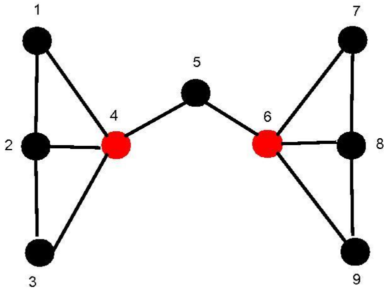

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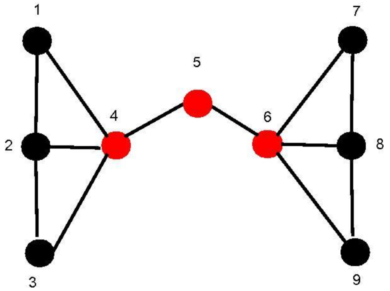

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- *NP*-hard
- Model for some telecommunications networks
- Models some circuit layout problems

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  Ex: Wireless Ad Hoc Networks ⇒ virtual backbone ⇒ MCDS
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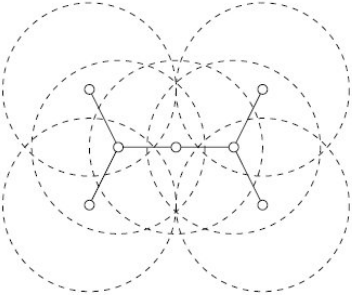

Previous Work

Garey and Johnson (1979): problem is *NP*-hard.

Lu and Ravi (1992,1998): factor of 3 approximation algorithm.

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There exists ϵ such that finding $(1+\epsilon)$-approximation algorithm is *NP*-hard

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Formulation of Fujie [2003,2004]

Variables involved:

- $x = \{x_e \in \mathbb{R}_+ : e \in E\}$: for the edges of G.
- $z = \{z_i \in \{0,1\} : i \in V\}$: to represent leaves of $\mathcal{T}$.

Notation:

- $\delta(i) \subseteq E$, $i \in V$: edges incident to i.
- $E(S) \subseteq E$, $S \subseteq V$: edges with both endpoints in S.

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\begin{aligned}
\max \sum_{i \in V} z_i & & \\
\text{s.t.} \sum_{e \in E} x_e &= |V| - 1 & \\
\sum_{e \in E(S)} x_e &\leq |S| - 1 & S \subset V, |S| \geq 2 \\
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Facet Defining Inequalities

Recall that inequality

$$\sum_{e \in \delta(i)} x_e + (|\delta(i)| - 1) z_i \leq |\delta(i)| \qquad i \in V$$

is used in formulation.

Then, under some mild assumptions - Fujie[2004],

$$\sum_{e \in F} x_e + (|F| - 1) z_i \leq |F| \qquad i \in V, F \subseteq \delta(i), |F| \geq 2$$

are facet defining inequalities for the polytope defined by the formulation.

Relaxation of Fujie's formulation

Formulation may be expressed as:

$$\max \sum_{i \in V} z_i$$

$$\text{s.t.} \quad \sum_{e \in \delta(i)} x_e + (|\delta(i)| - 1) z_i \leq |\delta(i)| \qquad i \in V$$

$$x \in ST_G$$

$$z_i \in \{0, 1\} \qquad i \in V$$

ST_G: set of all incidence vectors of spanning trees in G.

Relaxation:

- relax integrality constraints on z to $0 \leq z \leq 1$.

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Relaxation of Fujie's formulation (continuation)

Since one is maximizing $\sum_{i \in V} z_i$:

$$\sum_{e \in \delta(i)} x_e + (|\delta(i)| - 1) z_i \leq |\delta(i)|, i \in V,$$

must hold as equality at an optimal solution!

One may then write:

$$\begin{aligned} \max \quad & \sum_{i \in V} \frac{|\delta(i)| - \sum_{e \in \delta(i)} x_e}{|\delta(i)| - 1} \\ = \quad & \sum_{i \in V} \frac{|\delta(i)|}{|\delta(i)| - 1} - \sum_{e=(i,j) \in E} \left(\frac{1}{|\delta(i)| - 1} + \frac{1}{|\delta(j)| - 1} \right) x_e \\ \text{s.t.} \quad & x \in ST_G. \end{aligned}$$

- Relaxation is easy to solve: MSTP
- Relaxation bound equals LP-relaxation bound for original formulation.
- Branch-and-Bound algorithm based on solving MSTPs!

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$$\begin{aligned} \max \quad & \sum_{i \in V} \frac{|\delta(i)| - \sum_{e \in \delta(i)} x_e}{|\delta(i)| - 1} \\ = \quad & \sum_{i \in V} \frac{|\delta(i)|}{|\delta(i)| - 1} - \sum_{e=(i,j) \in E} \left(\frac{1}{|\delta(i)| - 1} + \frac{1}{|\delta(j)| - 1} \right) x_e \\ \text{s.t.} \quad & x \in ST_G. \end{aligned}$$

- Relaxation is easy to solve: MSTP
- Relaxation bound equals LP-relaxation bound for original formulation.
- Branch-and-Bound algorithm based on solving MSTPs!

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Relaxation of Fujie's formulation (continuation)

Since one is maximizing $\sum_{i \in V} z_i$:

$$\sum_{e \in \delta(i)} x_e + (|\delta(i)| - 1) z_i \leq |\delta(i)|, i \in V,$$

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Reformulation Strategies for MLSTP

Strategy 1: Directed graph reformulation
Max Leaf Spanning Arborescence Problem.

Strategy 2: Steiner Arborescence Problem on a Level Graph.

First MLSTP Reformulation

- Directed graph reformulation of Fujie's formulation: $x_e = y_{ij} + y_{ji}$, $e = [i,j] \in E$.
- $\delta^+(i)$ (resp. $\delta^-(i)$): arcs leaving (resp. entering) $i \in V$

$$
\begin{aligned}
\max \sum_{i \in V} z_i & \\
\text{s.t.} \sum_{a \in A} y_a = |V| - 1 & \\
\sum_{a \in \delta^-(j)} y_a = 1 & \qquad j \in V \setminus \{r\} \\
\sum_{a \in A(S)} y_a \leq |S| - 1 & \qquad S \subset V, |S| \geq 2 \\
\sum_{a \in \delta^-(i)} y_a + \sum_{a \in \delta^+(i)} y_a + (|\delta^+(i)| - 1) z_i \leq |\delta^+(i)| & \qquad i \in V \setminus \{r\} \\
\sum_{a \in \delta^+(r)} y_a + (|\delta^+(r)| - 1) z_r \leq |\delta^+(r)| & \\
0 \leq y_a \leq 1 & \qquad a \in A \\
z_i \in \{0, 1\} & \qquad i \in V
\end{aligned}
$$

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Advantage over Fujie's formulation

- Valid inequalities: $y_a + z_i \leq 1, \quad a = (i,j) \in A$
  no equivalent inequality in undirected formulation
- Since $x_e = y_{ij} + y_{ji}$, for $e = [i,j] \in E$, Fujie's valid inequalities are re-written as:

$$\sum_{a \in F} y_a + \sum_{a=(j,i) \in A | (i,j) \in F} y_a + (|F| - 1)z_i \leq |F|, \; i \in V, \quad F \subseteq \delta^+(i), \quad |F| \geq 2.$$

- Branch-and-Cut algorithm based on reinforced formulation!

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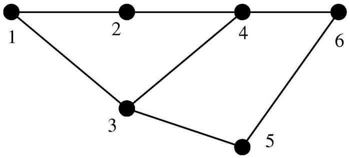
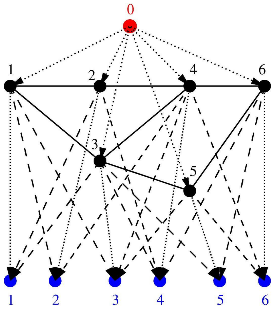
Second Reformulation for MLSTP

Steiner Arborescence Problem based on Level Graphs $D_F = (V_F, A_F)$

$$V_F = \{0\} \cup \{(i,h) : 1 \leq h \leq 2,\ i \in V\}$$
$$A_F = \{(0,(j,1)) : j \in V\} \cup \{((i,1),(j,1)) : [i,j] \in A\}$$
$$\cup \{((i,1),(i,2)) : i \in V\} \cup \{((i,1),(j,2)) : [i,j] \in A\}$$





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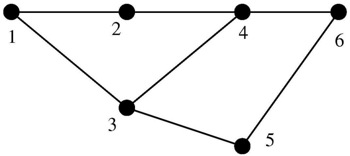

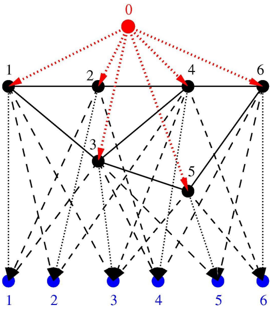

$c_{0j} \rightarrow \infty$ - only one arc $(0,(j,1))$

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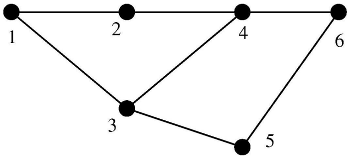

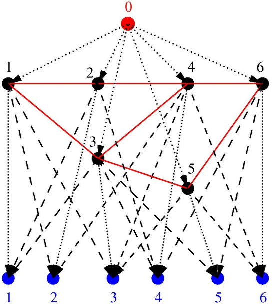

$c_{ij}^1 = |V| - 1$

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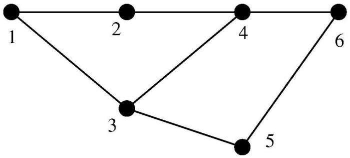

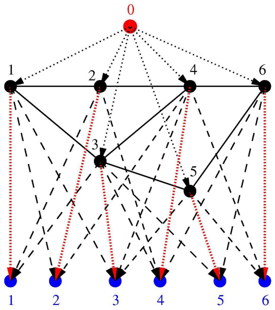

$c_{ii} = 0$

Second Reformulation for MLSTP

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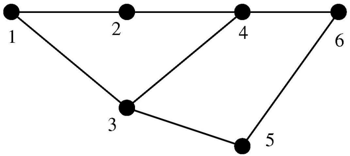
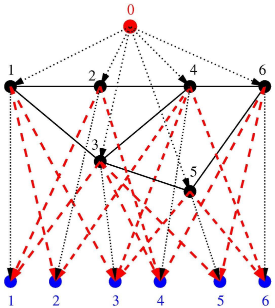
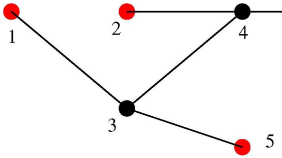


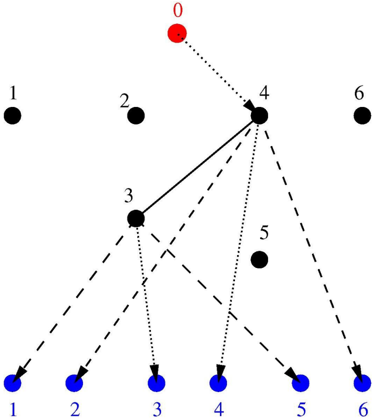


$c_{ij}^2 = 1$

Steiner Arborescence Reformulation for MLSTP

- Set of terminal vertices: $T = \{(i, 2) : \ i \in V\}$.
- Optimal solution for MLSTP in $G \Rightarrow$ Steiner tree in G_F, rooted at 0.
- Under conveniently defined costs, SAP reformulation for *MLSTP*.





Computational Results

|V|	dens.	opt.	B&B			Dir			SAP		
			LP bound	# nodes	T(s)	LP bound	# nodes	T(s)	LP bound	# nodes	T(s)
	10	15	17.83	291	0.12	15.57	1	0.01	15.80	1	0.04
	20	23	26.04	5055	0.33	24.48	7	0.1	23.95	1	0.12
30	30	26	27.44	842	0.24	27.05	1	0.03	26.13	5	26.7
	50	27	28.46	307	0.19	28.13	3	0.09	27.94	1	1.28
	70	28	28.83	1	0.16	28.73	1	0.01	28.00	1	0.26
	5	19	21.50	265	0.94	19.00	1	0.02	19.00	1	0.09
	10	38	42.16	82599	4.54	39.75	41	0.82	38.86	38	94
50	20	43	46.58	225771	16.9	45.22	77	1.32	44.48	57	1827
	30	45	47.59	38155	5.97	46.80	39	1.21	46.08	43	22424
	50	47	48.45	3050	4	48.18	13	0.51	47.36	-	-
	70	48	48.82	5	1.64	48.62	1	0.09	48.00	1	2.08
	5	43	49.57	9068999	313	44.45	53	0.99	43.56	19	103
	10	57	63.18	-	-	59.20	174	4.73	58.60	-	-
70	20	63	66.22	-	-	65.04	607	16.3	64.37	-	-
	30	65	67.62	4113677	536	66.91	35	2.9	66.15		-
	50	67	68.52	33058	25.3	68.14	7	1.33	-	-	-
	70	68	68.76	2661	10.8	68.68	5	1.92	68	1	8.55

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|V|	dens.	opt.	B&B			Dir			SAP		
			LP bound	# nodes	T(s)	LP bound	# nodes	T(s)	LP bound	# nodes	T(s)
100	5	76	83.42	-	-	79.36	605	24.5	78.57	-	-
	10	87	93.27	-	-	89.52	135	9.36	-	-	-
	20	92	96.56	-	-	94.85	1025	86.1	-	-	-
	30	94	97.39	-	-	96.68	1753	258	-	-	-
	50	96	98.36	348389	213	98.03	479	132	-	-	-
	70	97	98.76	9091	50.5	98.64	121	154	-	-	-
120	5	95	105.04	-	-	97.77	24	2.65	-	-	-
	10	107	113.16	-	-	109.83	869	65.4	-	-	-
	20	112	116.39	-	-	114.93	2401	393	-	-	-
	30	114	117.40	-	-	116.69	2301	653	-	-	-
	50	116	118.42	571335	435	118.12	1297	815	-	-	-
	70	117	118.72	13791	97	118.63	137	356	-	-	-
150	5	124	135.11	-	-	128.74	31077	2954	-	-	-
	10	136	142.81	-	-	139.59	6089	3247	-	-	-
	20	141	146.81	-	-	145.12	173425	61639	-	-	-
	30	144	147.38	-	-	146.67	3043	2617	-	-	-
	50	146	148.38	2104992	2190	148.10	1755	2756	-	-	-
	70	147	148.72	21625	301	148.63	219	1828	-	-	-
200	50	196	198.32	-	-	198.07	3125	20155	-	-	-
	70	197	198.73	38215	1322	198.63	253	8154	-	-	-

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Conclusions

Fujie's algorithm:

- Weaker upper (LP) bounds.
- Demanding in terms of memory.
- Faster for some high density instances.

Directed graph reformulation:

- Stronger LP bounds than original undirected formulation.
- Faster in 31 out of 37 instances tested.
- Attempted to solve larger number of test instances.

SAP reformulation:

- Stronger LP bounds than other formulations.
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