

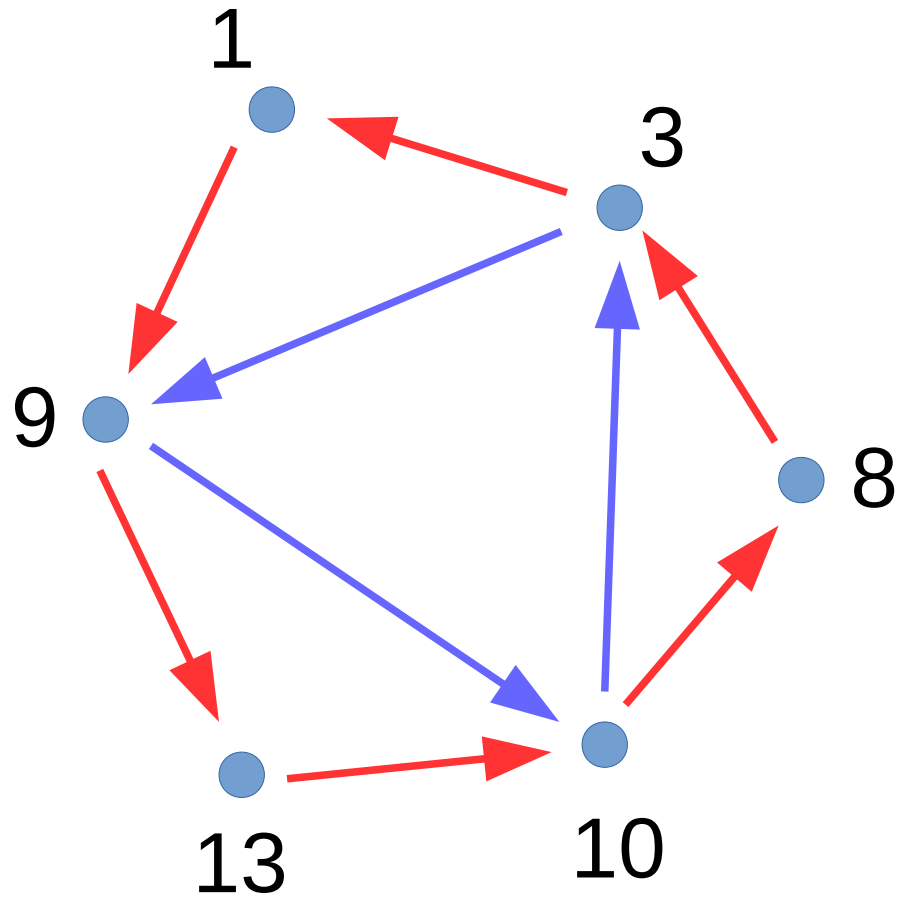
# Algebraic distance

- Definition: weight of a lightest series of operations that transform genome  $\pi$  into genome  $\sigma$
- Recall:  $\rho$  is applicable to  $\pi$  when  $\pi\rho$  is a genome
- Recall: weight of  $\rho = \|\rho\|/2$
- $d_{\text{alg}}(\pi, \sigma) = \|\sigma\pi^{-1}\|/2 = \|\pi\sigma^{-1}\|/2 = \|\pi^{-1}\sigma\|/2 = \|\sigma^{-1}\pi\|/2$
- $d_{\text{alg}}(\pi, \sigma) = \|\sigma\pi\|/2$  in adjacency algebraic

# Finding sorting operations

- Compute  $\sigma\pi^{-1}$
- Find a cycle  $\mu$  dividing  $\sigma\pi^{-1}$
- If  $\mu$  and  $\pi\mu^{-1}\pi^{-1}$  are disjoint, then  $\rho = \mu\pi\mu^{-1}\pi^{-1}$  is a sorting operation on  $\pi$

# Cycles dividing cycles



# Weights of classical operations

- Reversals, translocations, circular fusions, circular fissions: 2-breaks
- 2-breaks are of the form  $\mu\pi\mu^{-1}\pi^{-1}$  for  $\mu$  a 2-cycle
- 2-breaks have weight = 1
- Linear fusions, linear fissions, circularizations, linearizations: weight = 1/2

# Relationship with DCJ

- Adjacency graph  $AG(\pi, \sigma)$
- $d_{\text{alg}} = N - C - P/2$
- $d_{\text{DCJ}} = N - C - P_{\text{odd}}/2$
- Warning:  $P_{\text{odd}}$  in  $AG$  is not the same as  $P_{\text{odd}}$  in  $BG$  (breakpoint graph)

# Linear chromosomes

- Chromosomal algebraic representation
- Circular chromosomes:
  - two cycles
  - each one is the reverse complement of the other
- Linear chromosomes:
  - one cycle
  - the cycle is self-reverse-complementary

# Algebraic structure of operations

- Linear fission / linearization:  $\rho = (-u \ v)$ 
  - $u, v$  consecutive blocks
- Linear fusion / circularization:  $\rho = (u \ v)$ 
  - $u, v$  telomeres being linked

# Algebraic structure of operations

- Reversal:  $\rho = (-u -v)(w x)$ 
  - $v, w$  consecutive blocks;  $u, x$  consecutive blocks
  - same chromosome, different strands
- Circular fission / excision:  $\rho = (-u -v)(w x)$ 
  - $v, w$  consecutive blocks;  $u, x$  consecutive blocks
  - same chromosome, same strand



# Algebraic structure of operations

- Circular fusion / translocation:  $\rho = (-u -v)(w x)$ 
  - v, w consecutive blocks; u, x consecutive blocks
  - different chromosomes, same type
- Circular reabsorption:  $\rho = (-u -v)(w x)$ 
  - v, w consecutive blocks; u, x consecutive blocks
  - different chromosomes, different type

# Algebraic structure of operations

- Transposition:  $\rho = (-u -v -w)(x y z)$ 
  - $u, z; v, y; w, x$  consecutive blocks
  - same strand, same order
- Block interchange:  $\rho = (-u -v)(w x)(-y -z)(p q)$ 
  - $v, w; y, q; u, x; z, p$  consecutive blocks
  - same strand, same order