## Matrix Medians

- Permutations can be seen as matrices
- Norm of $A=$ rank of $A-I$
- Median of matrices $A, B, C$ : M such that
$\operatorname{rank}(A-M)+\operatorname{rank}(B-M)+\operatorname{rank}(C-M)$
is mimimum


## Finding medians

- Small rank(A - M)
- Large vector space where A = M
- The same must happen with $B$ and $C$


## Spaces where matrices agree

$A=B$

$$
B=C
$$

$$
A=C
$$

## Decomposing $\mathrm{R}^{n}$

$k_{5} \quad V_{*}$ (.A.B.C. $) \quad \mathrm{M}=\mathrm{A}, \mathrm{B}$, or C

| $k_{3}$ | $k_{2}$ | $k_{4}$ |
| :---: | :---: | :---: |
| $\mathrm{~V}_{\star}(. \mathrm{A} . \mathrm{BC})$. | $\mathrm{V}_{\star}\left(. \mathrm{AB} . \mathrm{C}_{.}\right)$ | $\mathrm{V}_{\star}(. \mathrm{AC} . \mathrm{B})$. |
| $\mathrm{M}=\mathrm{B}$ | $\mathrm{M}=\mathrm{A}$ | $\mathrm{M}=\mathrm{A}$ |

$k_{1}$
$V_{*}$ (.ABC.)
$M=A$

$$
k_{1}+k_{2}+k_{3}+k_{4}+k_{5}=n
$$

## Median approximations

- $\mathrm{M}_{\mathrm{A}}=\mathrm{A}$ in $\mathrm{V}_{*}$ (.A.B.C.)
- $M_{B}, M_{C}$ are defined similarly
- $\mathrm{M}_{\mathrm{A}}, \mathrm{M}_{\mathrm{B}}, \mathrm{M}_{\mathrm{C}}$ are approximations to a median
$d\left(M_{x} ; A, B, C\right) \leq 4 / 3$ median score

