PQR-Trees

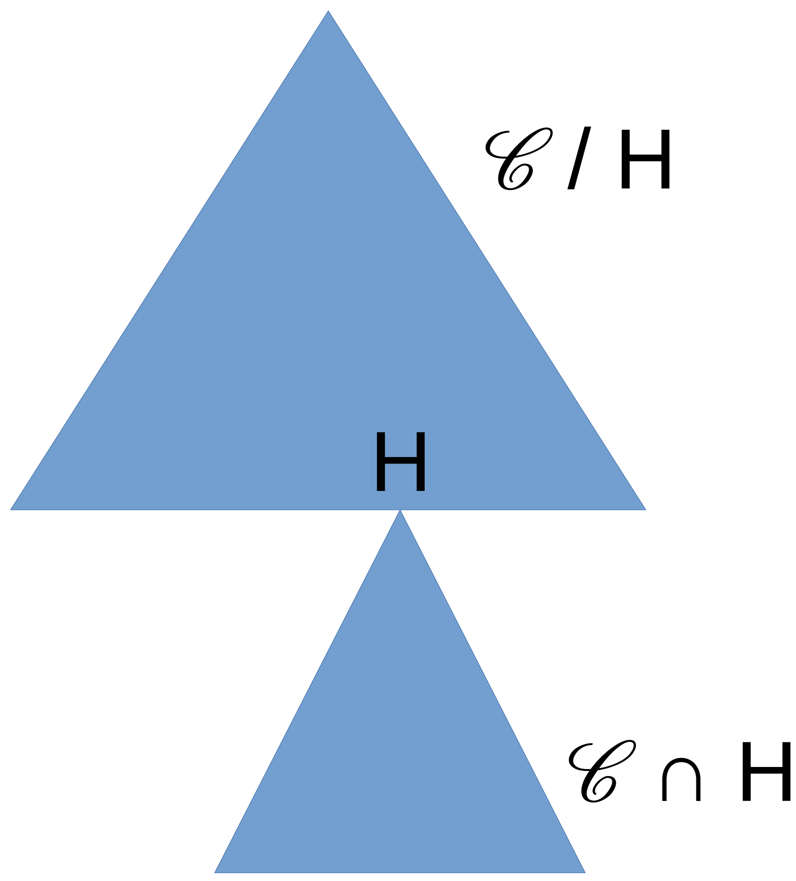
- PQR-Trees: generalize PQ-Trees
- Offline, $O(n^2)$ algorithm
- Divide-and-conquer methodology

- Input: $(U, \mathscr{C})$
- $U = \{a_1, a_2, \ldots, a_m\}$
- $\mathscr{C} = \{S_1, S_2, \ldots, S_n\}$, $S_i \subseteq U$

Divide-and-conquer

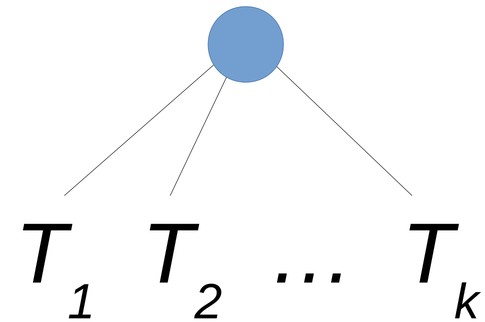
- Find a suitable set H
- Divide problem into two subproblems:
  - (H, $\mathscr{C} \cap$ H)
  - (U/H, $\mathscr{C}$ / H)
- $\mathscr{C} \cap$ H: sets contained in H
- $\mathscr{C}$ / H:
  - sets disjoint with H and
  - sets containing H, with subset H replaced by a "super-element"

Joining solutions

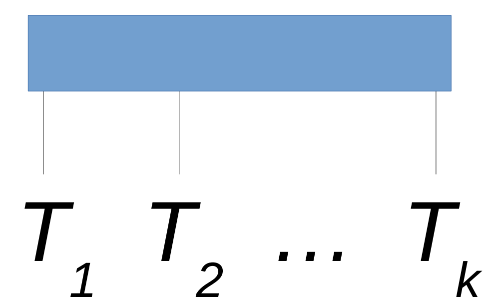


PQR-Trees

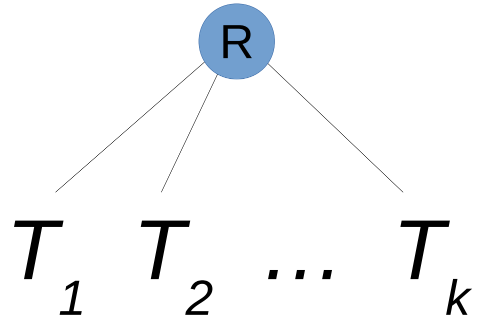
- Universal set U
- Leaves: elements of U
- Internal nodes:
  - P nodes



  - Q nodes



  - R nodes



How to find H

- Nontrivial union of component in strictly overlapping graph
- If all unions of components are trivial, nontrivial twin class
- If all unions of components are trivial, and twin classes are trivial: **Prime collection**

- **Trivial sets:** singletons $\{a_i\}$, U

Prime collections

- Prime collections are the base cases for divide-and-conquer
- There are only three types:
  - Trivial sets only: P node
  - Sets compatible with linear order: Q node
  - All other cases: R node