CHAPTER 3 ❑ MATLAB EXERCISES

1. Use MATLAB to calculate the determinants of the following matrices.

(a) $\begin{bmatrix} 2 & -3 \\ -6 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} -5 & 6 & 7 \\ 0 & -1 & 2 \\ 4 & 0 & -3 \end{bmatrix}$

(c) **pascal**(**4**) (d) **hilb**(**8**)

2. Let

$$A = \begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix}.$$

Use the MATLAB determinant command **det** to compute **det**(**2** $*$ **eye**(2) − **A**)**.** Find an integer value of t such that $\det(tI - A) = 0$.

3. Choose arbitrary 4×4 matrices A and B. Compute $\det(A)\det(B)$, and $\det(AB)$. What do you observe? Do the same for $\det(A) + \det(B)$ and $\det(A + B)$.

4. Choose an arbitrary real number t. Form the matrix

$$A = \begin{bmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{bmatrix}$$

and calculate its determinant. Does the value of the determinant depend on t?

5. Consider the matrices

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & -1 & 4 \\ 0 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}.$$

(a) Verify that $\det(A)\det(B) = \det(AB)$.
(b) Verify that $\det(A^T) = \det(A)$.
(c) Verify that $\det(A^{-1}) = 1/\det(A)$.

6. In this exercise, we will use Cramer's Rule to solve the linear system $A\mathbf{x} = \mathbf{b}$ from Example 3, Section 3.4. Let

$$A = \begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 10 \\ 11 \end{bmatrix}$$

be the coefficient matrix and the right-hand side, respectively. To form the matrix A_1, we need to replace the first column of A with **b.** To do this, type

A1 = A
A1(:,[1]) = b

The solution x_1 is obtained by typing

det(A1)/det(A)

You can calculate x_2 in a similar manner.

A2 = A
A2(:,[2]) = b
det(A2)/det(A)

7. Use the Cramer's Rule algorithm from Exercise 6 to solve the following linear system. Compare your answer with that obtained using **rref.**

$$\begin{aligned} 3x + 3y + 4z &= 2 \\ x + y + 4z &= -2 \\ 2x + 5y + 4z &= 3 \end{aligned}$$