CHAPTER 5 ❑ MATLAB EXERCISES

1. Use the MATLAB command **norm(v)** to find

(a) the length of the vector $\mathbf{v} = (0, -2, 1, 4, -2)$.

(b) a unit vector in the direction of $\mathbf{v} = (-3, 2, 4, -5, 0, 1)$.

(c) the distance between the vectors $\mathbf{u} = (0, 2, 2, -3)$ and $\mathbf{v} = (-4, 7, 10, 1)$.

2. The dot product of the vectors (written as columns)

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

is given by the matrix product

$$\mathbf{u} \cdot \mathbf{v} = \mathbf{u}^T\mathbf{v} = [u_1 \quad u_2 \quad \ldots \quad u_n]\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}.$$

Hence you can compute the dot product of **u** and **v** simply by multiplying the transpose of **u** times the vector **v**. Let $\mathbf{u} = (2, -5, 0, 4, 8)$, $\mathbf{v} = (0, -3, 2, -1, 1)$ and $\mathbf{w} = (1, -1, 0, 0, 7)$, and use MATLAB to find the following.

(a) $\mathbf{u} \cdot \mathbf{v}$

(b) $(\mathbf{u} \cdot \mathbf{v})\mathbf{w}$

(c) $\mathbf{u} \cdot (2\mathbf{v} - 3\mathbf{w})$

(d) $\mathbf{v} \cdot \mathbf{v}$ and $\|\mathbf{v}\|^2$

3. The angle θ between two nonzero vectors is given by

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}.$$

Use MATLAB to find the angle between $\mathbf{u} = (-3, 4, 0)$ and $\mathbf{v} = (1, 1, 4)$. (Hint: use the built-in inverse cosine function, **acos**).

4. You can find the orthogonal projection of the column vector **x** onto the vector **y** by computing

$$\frac{\mathbf{x}^T\mathbf{y}}{\mathbf{y}^T\mathbf{y}}\mathbf{y}.$$

Use MATLAB to find the following projections of **x** onto **y**.

(a) $\mathbf{x} = (3, 1, 2)$, $\mathbf{y} = (7, 1, -2)$

(b) $\mathbf{x} = (1, 1, 1)$, $\mathbf{y} = (-1, 1, 1)$

(c) $\mathbf{x} = (0, 1, 3, -3)$, $\mathbf{y} = (4, 0, 0, 1)$

5. Use the MATLAB command **cross(u, v)** to find the cross products of the following vectors.

(a) $\mathbf{u} = (1, -2, 1)$, $\mathbf{v} = (3, 1, -2)$

(b) $\mathbf{u} = (0, 1, -2)$, $\mathbf{v} = (-5, 14, 6)$

6. Let $\mathbf{u} = (-3, 2, 4)$, $\mathbf{v} = (5, 0, -7)$, and $\mathbf{w} = (-1, -5, 6)$. Use MATLAB to illustrate the following properties of the cross product.

(a) $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$

(b) $\mathbf{u} \times \mathbf{u} = \mathbf{0}$

(c) $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$

(d) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$

7. The MATLAB command **A\b** finds the least squares solution to the linear system of equations $A\mathbf{x} = \mathbf{b}$. For example, if

$$A = \begin{bmatrix} 0 & 2 \\ 3 & 0 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix},$$

then the command **A\b** gives the answer

$$\begin{bmatrix} 0.6000 \\ 0.5000 \end{bmatrix}.$$

Use MATLAB to solve the least squares problem $A\mathbf{x} = \mathbf{b}$ for the given matrices.

(a) $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 3 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} -1 \\ 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

8. Use MATLAB to find the four fundamental subspaces of the following matrices.

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 1 & 4 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 3 & -2 & -1 & 0 \\ 4 & 1 & -1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$