CHAPTER 2 ❑ MATLAB EXERCISES

1. Enter the matrices

$$A = \begin{bmatrix} 0 & -4 & 5 \\ 3 & 1 & -2 \\ 2 & 1 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -5 & 6 & 7 \\ 0 & -1 & 2 \\ 4 & 0 & -3 \end{bmatrix}.$$

Use MATLAB to find

(a) $A + B$ (b) $B - 3A$ (c) AB (d) BA.

2. Enter the three matrices

$$A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{bmatrix} \quad B = \begin{bmatrix} 1.0000 & 0.5000 & 0.3333 & 0.2500 \\ 0.5000 & 0.3333 & 0.2500 & 0.2000 \\ 0.3333 & 0.2500 & 0.2000 & 0.1667 \\ 0.2500 & 0.2000 & 0.1667 & 0.1429 \end{bmatrix}$$

$$C = \begin{bmatrix} 16 & -120 & 240 & -140 \\ -120 & 1200 & -2700 & 1680 \\ 240 & -2700 & 6480 & -4200 \\ -140 & 1680 & -4200 & 2800 \end{bmatrix}.$$

(a) Calculate $A - B$. Use the **format long** command to verify the result. Return to the standard short format with **format short.**

(b) Calculate AC and BC. Define what is meant by the inverse of a square matrix. What is the inverse of the matrix A? Of the matrix C?

3. Write the following system of linear equations in the form $AX = B$ and use the MATLAB command **A\ B** to solve the system.

$$\begin{aligned} 3x + 3y + 4z &= 2 \\ x + y + 4z &= -2 \\ 2x + 5y + 4z &= 3 \end{aligned}$$

Check your answer using **rref.**

4. Enter the matrices

$$A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 2 & -1 & -5 \\ 7 & -5 & 0 & 6 \\ -4 & 0 & 7 & 12 \end{bmatrix} \quad \text{and} \quad B = \textbf{pascal(4)}.$$

(a) Use the MATLAB command **trace** to find the traces of A, B, and $A + B$. What do you observe?

(b) What is the relationship between the trace of AB and the trace of BA?

5. Use the MATLAB command **diag** to form the 5×5 diagonal matrix D with diagonal entries 0, -1, -2, -3, and -4. Find the product $D^4 = DDDD$. If D is an $n \times n$ diagonal matrix, describe how to find the product D^k for any positive integer k.

6. Enter the three matrices

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & -6 & 3 & 5 \\ 0 & -3 & -6 & 8 \\ 3 & 5 & 0 & 7 \\ -1 & 0 & -7 & 9 \end{bmatrix} \qquad C = \begin{bmatrix} 16 & -1 & 4 & -1 \\ -3 & 12 & -7 & 8 \\ 4 & -5 & 0 & 0 \\ -14 & 3 & 2 & 8 \end{bmatrix}$$

(a) Calculate $AB - AC$ and $A(B - C)$. What do you observe?
(b) Calculate $3(AC)$, $A(3C)$, and $(3A)C$. What do you observe?

7. Let

$$A = \begin{bmatrix} 1 & \frac{1}{3} \\ 0 & \frac{1}{4} \end{bmatrix}.$$

Compute A^2, A^3, and A^8. Describe the matrix A^n for large n.

8. Enter the matrices

$$A11 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad A12 = \textbf{zeros(2, 2)} \qquad A22 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

(a) Form the 4×4 matrix A using the following MATLAB construction:
A = [A11 A12; A12 A22]
(b) Find the smallest value of n such that $A^n = A$.

9. Use the MATLAB command **inv** to find the inverse of the following matrix A. Then adjoin the identity matrix $I =$ **eye(3)** to A to form the 3×6 matrix $B = [A \quad I]$. Row-reduce B to compute the inverse of A again. What do you observe?

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ 7 & 16 & -21 \end{bmatrix}$$

10. Let A and B be the following 3×3 matrices.

$$A = \begin{bmatrix} 2 & 4 & \frac{5}{2} \\ -\frac{3}{4} & 2 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{4} \\ \frac{3}{2} & \frac{1}{2} & -2 \\ \frac{1}{4} & 1 & \frac{1}{2} \end{bmatrix}$$

(a) Calculate $A^{-1}B^{-1}$, $(AB)^{-1}$, and $(BA)^{-1}$. What do you observe?
(b) Find $(A^{-1})^T$ and $(A^T)^{-1}$. What do you observe? Remember: The MATLAB command for the transpose of a matrix A is A'.

11. In this project you will use MATLAB to find the least squares regression line for the set of data (1, 1), (2, 2), (3, 4), (4, 4), and (5, 6) from Example 10, Section 2.5.

(a) Form the following matrices.

$$X = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} \qquad Y = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 4 \\ 6 \end{bmatrix}$$

(b) Let $A = (X^TX)^{-1}X^TY$.
(c) Compare your answer with the MATLAB least squares command **polyfit.** (Hint: Let $X1 = X(:, 2)$ and type **polyfit(X1,Y,1).**)

(d) Plot the data using the MATLAB commands

```
t = (0: 0.1: 6);
plot(X1,Y, '+')
```

(e) Plot the least squares line for these data by using the MATLAB commands

```
t = (0: 0.1: 6);
p=polyfit(X1,Y,1);
f=polyval(p,t);
plot(t,f, '*')
```

(f) You can combine the previous plots as follows

```
plot(X1,Y, '+',t,f, '*')
```

12. Repeat Exercise 11 for the data $(0, 6)$, $(4, 3)$, $(5, 0)$, $(8, -4)$, $(10, -5)$. Plot the data and the least squares line on the interval $[0, 10]$. That is, use **t = (0: 0.1: 10);.**