ASPECTS FROM THE HISTORY OF GEOMETRY

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Abstract. The paper presents some historical aspects from the geometry, especially from descriptive geometry.

1. THEORETICAL CONSIDERATIONS

The necessity to represent the objects in the space by drawings – means others than the voice – has appeared from the ancient times and from this point of view, the origin of the elementary notions of geometry cannot be precisely determined.

The first graphical representations of the primitive man, performed on the cave walls with simple tools, had the purpose to transmit messages to the fellows and revealed aspects from the social life and from nature. These drawings, expressing the visual perceptions of the environment, performed as simple intuitive representations, are inspired from the process of the image formation in the human eye.

After successive perfectioning, people have started to represent the objects from their own imagination, through the plastic arts (graphic, artistic drawing, painting), in order to transmit to their fellows an aesthetic emotion, or in order to precisely represent the objects they wish to build, through specific graphical methods.

The graphical representation of the objects by drawings has been imposed by the construction of the tools, guns, roads, bridges, fortifications, buildings, as well as of all the necessary material objects.

The technique of building has led to the representation of the different parts or details of the building, through some plane drawings. After these drawings, the builders precisely carve the stones, with minimal loss, such as the stone to be placed in the building as precisely as possible. These drawings and principles of representation were considered by the builders as professional secrets and were transmitted only to their pupils.

The man, in analogy with the physiological process of the vision – in which the three-dimensional objects in the space are perceived as a bidimensional image on the retina – has imagined different methods to represent on a plane, an object in the space. Of all the elaborated methods of representation, only the practical ones have perfected and remained in use.

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2. SHORT HISTORY OF THE GEOMETRY

The mathematic has developed having as starting points, two fundamental elements: the size and the shape.

The size has led to the concept of number and the elementary operations with numbers have generated the arithmetic, the theory of numbers and, subsequently, the algebra. The shape of the objects in nature and their measurement have led to the foundation of the geometry. Later, the causality, that has a certain correspondence in the notion of function, has developed the mathematical analysis.

The pictography, the oldest type of writing, dates since 15.000–20.000 years and fundamented all the known writings. The first material to write was the stone, natural or encastrated in buildings. The old pictographical signs have slowly became ideograms (in Greek: idea – idea and gramma – sign), meaning signs expressing ideas, notions, and not only the represented object, as in the pictographic writing (T. René, 1970).

The first writing symbols on clay plates, combining pictograms with abstract symbols – associated with the Vinca culture – appeared 4.500 years B.C. at Tărtăria, Alba county, Romania.

The first documents written on clay plates, in cuneiform writing (in Latin: cuneus – nail) appeared in Mesopotamia (in Greek: "The country between rivers"), 3.500 years B.C. The cuneiform writing had, at the beginning, 891 signs and was generated by the necessity to contabilize the material goods from commerce.

The Sumerians used to be a small people, occupying a modest territory on the shore of the Persia Gulf, along the inferior course of the Euphrates. The Sumerians invented the first writing system, unitary and coherent, as well as the first texts, but they were unable to create an alphabet.

The Babylonians have been the inventors of the algebra and were "calculators" in the true meaning of the word. Because of this, they had the tradition to transpose all the relations in numerical language and to algebrize the purely geometrical problems. The Babylonian numeration has been a positional numeration having a sexagesimal base. The Babylonian geometrical knowledge consisted in the use of some simple geometrical relations, isolated from any demonstrative context and used in pedagogical purposes. The Mesopotamian science has been a science of study, formation and application.

The Egyptians, 2.800 years B.C. have been using a decimal, hieroglyphic numeration system, but they did not know the symbol „zero". The Egyptian writing has evoluated in three phases, named hieroglyphical, meaning "of the fine notches" (in Greek: hieros – saint and glypho – cut), hieratic, meaning of the "priests" (in Greek: hieratikos – of the priests) and demotic, meaning popular (in Greek: demos – people). The most valuable material on which the Egyptians used to write was the papyrus.

The famous Greek historian Herodot (485–425 B.C.) attributes the invention of the geometry to the Egyptians, which the Greeks have moved in their country,

because of the necessity to measure precisely the surface of the lands, in order to establish the taxes. The Egyptian geometry, having an elementary level, was not based on reason; its utilitarian character imposed the obtaining of solutions by approximations. The Egyptians have magistrally used the elementary geometry in order to reach perfection and rigurosity in the art of constructions.

In the island of Crete, around 1750 B.C., the Linear A writing has been elaborated, having 95 ideograms, then between 1450–1400 B.C., the Linear A writing has been replaced with the Linear B one, formed by 88 signs.

In India, starting with the year 1.500 B.C., has started the elaboration of the "Veda" or "The Science", written in the old Sanskrit, including the antique Indian science. The Indians have invented and introduced in practice the complete decimal numeration system, with 9 figures and with "zero" (that became universal by its propagation, in the whole world, by the Arabs). The Indian geometry included simple books of technical rules, without demonstrations (A.Şt., 1965).

China, starting the IIIrd millennium B.C., has developed, mainly, in the basin of the Huang-He river. In the XIVth century B.C., the Chinese writing (the only ideographical writing still in use today) has been invented and the expression of numbers and decimals were made analytically and decimally. The Chinese geometry was elementary and had a practical destination.

At the half of the IInd millennium B.C., in the Syrian-Palestinian region have been present two forms of alphabet: the cuneiform alphabet from Ugarit (Syria) – having 30 signs – and the linear Phoenician one. The Phoenician alphabet had 22 signs and the writing was realized horizontally, from the right to the left. The Phoenician alphabet has the merit to sit at the base of all the occidental alphabets. The invention of the alphabet has also contributed to the spread of the papyrus.

In the Middle East, from the Phoenician alphabet, the Paleo-Hebraic, Armenian (a language in the north of Judea and Syria) and new Hebraic writings have developed. (VIth century B.C.).

The elaboration and adoption, in Greece, of the Greek alphabet, based on the perfection of the Phoenician alphabet, has been noted at the beginning of the IXth century B.C.

The Etruscan alphabet appeared towards the years 700 B.C. and has been an adaptation of the Greek alphabet to the Etruscan language.

The Latin alphabet, used by the Romans to express the Latin language, had at its base either the Etruscan, or the Greek alphabet (from the Greeks living in Cuma, on the west shore of the Italic peninsula).

The Hellenic science has developed on a three centuries interval (VIth B.C.–IVth B.C.) and distinguished by originality and superiority from all that the humankind had thought and created so far.

The Greeks have realized remarkable progresses, both in the thinking methods and in the amplitude of knowledge. The mathematics based on demonstration has been a creation of the Hellenic science (B. Nicolae, 1981).

Tales from Milet (624–547 B.C.), one of the "seven wise men of the antique Greece", has been considered "the parent of the Greek science", as well as the first Greek mathematician. In Egypt, Tales has measured the height of the Pyramid of Kheops by the help of the shadows, as well as the distance from a tower to the ships located on the sea, using the proportionality of the sides of an alike triangle. Tales has introduced the demonstration.

Pitagora from Samos (580–500 B.C.) and his pupils have introduced in geometry the abstractization, the precise definitions and the rigorous demonstrations. Pitagora has discovered the irrational numbers, the construction of the regular polyhedrons and the theorem with his name.

Zenon from Elea (495–430 B.C.) has introduced the method of absurd reduction.

Hipocrate from Kios (Vth century B.C.) has built the curvilinear figures named lunules.

Democrit (460–370 B.C.) has introduced the notion of cylinder.

Platon (427–347 B.C.) has founded in Athens, in 388 B.C., a school named Academia whose activity will last up to 529 and on whose front door was written: "the one who does not know the geometry, should not enter here". Platon has introduced the notions of: hypotenuse, diagonals, diameter and pyramid (D. Ovidiu, 1997).

Eudoxus from Knidos (408–355 B.C.) has elaborated the exhaustive method and put the bases of the sizes and reports theories, differentiating the commensurable from the incommensurable sizes.

Aristotel from Stagira (384–322 B.C.) knew the cycloid, has introduced the term of axiom and has used for the first time the letters of the alphabet to note the sizes.

Menechmus (375–325 B.C.) has discovered the conical sections.

The problem of the circle quadrature (the calculation of the number pi), the doubling of the cube and the angle trisection have been the three classical unsolved problems of the Greek mathematic. These problems have been definitively resolved by the mathematicians in the XIXth century.

The Hellenistic and Roman science has developed on a period comprised between the death of Alexander the Great in 323 B.C. and the official ending of the West Roman Empire, in 476.

The capital of the Hellenistic civilization has been Alexandria, the biggest Greek city in Egypt, founded by Alexander the Great, having two important cultural institutions: the Museum and the Library. The Library used to be a unique informational source, by the 700.000 roles of papyrus and parchment.

The Hellenistic geometers are credited with the invention of the trigonometry. The Greek numeration used two systems: attic (herodian) and a written, demipositional and decimal numeration.

Euclid (320–270 B.C.) has published "The Elements", fundamental writing, in 13 volumes, that has dominated the elementary mathematic up to the past

century. Euclid started from a small number of simple propositions, that are verified in nature and that were named postulates and axioms. The rest of the propositions are logically demonstrated, on the basis of the axioms, and are named theorems. Here it has been initiated the tradition to indicate the ending of a demonstration by the words: quod erat demonstrandum (what was to be demonstrated). The famous postulate of Euclid has been the first historical proof of a specifically mathematical attitude.

Archimedes from Syracuse (287–212 B.C.) has introduced the notions: demicircle and perimeter. He also studied: the quadrature of the parabola, the rapports between the sphere and the cylinder, the conoids and the spheroids. Archimedes has discovered the spiral with his name and the demiregular polyhedrons.

Apollonius from Perga (260–200 B.C.) has published the "Konika", made from 8 books, in which he has studied the conics from the algebraic point of view. Apollonius has studied for the first time: the inversion, the homothetia, the circular helix, the stereographical projection of the sphere on a plan, as well as the introduction of the following names: ellipse, hyperbola, parabola and directrix of a conic.

Nicomede (the IIIrd century B.C.–the IInd century B.C.) has discovered, in 200 B.C., the conchoid with his name.

Diocles (240–180 B.C.) has discovered, in 180 B.C., the cysoid (bearing his name), its name coming from the likeliness with an ivy leaf.

Perseu (IInd century B.C.) has studied, in 150 B.C., the spyrices (bearing his name).

Menelau from Alexandria (70–103) has published the book "Sferika", including geometry and spherical trigonometry, has introduced the theorem bearing his name and has defined the spherical triangle.

Claudiu Ptolemeu (75–165) has introduced, in his paper „Megale sintaxis", the division of the circle in 360 parts and of the sexagesimal degrees in minutes and seconds.

Heron from Alexandria (centuries I–II), in his work "Metrica", has described problems of applied mathematics.

Diofant from Alexandria (the IIIrd–IVth centuries), in his work "Arithmetica", in 13 books, he conceived an arithmetic in which the number has been considered abstract, thus becoming a precursor of the algebra.

Pappus from Alexandria (the IIIrd century), in his work "Collections" has introduced the surface named helicoid, as well as the notions of: anarmonic rapport and complete quadrilateral.

Eumen the IInd, king of Pergam (from Minor Asia) between 197–159 B.C., has discovered the parchment, that has completely replaced the papyrus in book kraft only in the IVth century.

In the pre-Columbian America, an advanced stage of the mathematics has been encountered in the civilized people of the Inca Empire and from the Mesoamerican

zone. The Incas had a decimal numerical system. The people from the mesoamerican zone (Aztec and Maya) have used a numerical system with the base 20. The old Mayas have used the pictographical and hieroglyphical writings.

In 105 A.C., the Chinese Tshai Lun has invented the paper. The Arabs took this technology in the VIIIth century and introduced it in the Occident. In 115, the Chinese have invented the writing with ink and panel.

The Arab alphabet, based on the perfectioning of the Arameic-Syrian alphabet, has formed in the VIth century, but a fully developed writing has been identified only after the VIIth century.

The Arab science has been known mainly in the Arab language and all the people that had been conquered by the Arabs brought their contribution to this science and all those who came into contact with the Islam have influenced it. Therefore, it can be said that the Arab science has been, as a matter of fact, a continuation of the Greek science.

Baghdad has been the first important scientific centre of the Arab caliphate in which, between the VIIIth–the IXth centuries, research in the field of the exact sciences were performed.

The Indian mathematician Brahmagupta (598–665) has introduced the sign zero (in Sanskrit: shunya – empty).

The Arabs adopted the complete decimal numerical system, with nine figures and zero, that had been previously introduced in India, after its publishing by the mathematician Al-Khwarizmi. The spread and perfectioning of the decimal arithmetic, based on the principle of the position value of the symbols, represent a great achievement of the Arab science. The Arab mathematics has developed the geometry, mainly the theory of the parallels, and especially the algebra and the trigonometry, that have constituted for the first time, as autonomous sciences.

Mohammed Ibn Musa Al-Khwarizmi (780–850), (the word algorithm is derived from his name), the creator of the algebra, has published, in 825, the book "Al-jabr wal muqabalah" (Transposition and reduction) – the word (Al-jabr) being at the origin of the term algebra. At the same time, he introduced zero (in Arab: sifr – empty), that is at the origin of the European word figure. The Indian figures are named, since then, "Arabic".

The European scientists have rediscovered the antic and Arabic science, thanks to the translations in Arab from Spain.

Gerbert D'Aurillac (940–1003) has been the first scientist that popularized in Europe, the Arabic figures.

Gerardo from Cremona (1114–1187) has introduced in 1150 the name of sinus.

Leonardo da Pisa (1170–1240), also named Fibonacci has published, in 1200, "Practica geometriae", the first original book on geometry in Europe. Fibonacci has introduced, in 1228, the row with his name, and in his paper "Liber abaci

apar“, appear for the first time, the notions: line of fraction, negative number and equation.

At the beginning of the XIIIth century, the European universities have been founded.

Bradvardinus Thomas (1290–1349) has introduced, in Europe, the notion of trigonometric circle.

Oresme Nicolas (1320–1382) has introduced, in 1370, the notion of rational number.

The invention of the printing press by Johann Gensfleische von Gutenberg from Mainz (1400–1468), in 1454, allowed the spread of the science. In mathematics it has continued the assimilation of the medieval and Arabic science, accompanied by the retyping of the elaboration of new manuals in the Latin language and in the national languages.

Starting with the XVth century, different Italian artists, like: Filippo Brunelleschi (1377–1446), Lorenzo Ghiberti (1378–1455), Leon Battista Alberti (1404–1472), Piero della Francesca (1404–1492), Leonardo da Vinci (1452–1519) and Benvenuto Cellini (1500–1571) put the bases of the perspective theory.

Leonardo da Vinci (1452–1519) has introduced the term of gold section.

Johann Widman (1460–1500) has introduced, in Leipzig, in 1489, the sign plus (+) and the sign minus (–) in his work “Rapid and confortable calculus for all the traders.”

Johann Werner (1468–1528) from Nürnberg has published, in 1522, the first original study about the conics that appeared in the Occident with the title “Libellus super vigintiduobus elementis conicis”.

The painter Albrecht Dürer (1471–1528) has published, in 1525, in Nürnberg, its work “Underweysung der Messung mit dem Zircked und Richtscheyt” in which it has been described, for the first time, the method of the double orthogonal projection. This study has been subsequently translated into Latin. Thus, A. Dürer has become a precursor of the Descriptive Geometry by Gaspard Monge. A. Dürer has introduced, in 1525, as ornamental figures, the epicycloids.

Michael Stifel (1487–1567) has introduced the term exponent.

Pedro Nunes (1492–1577) has introduced, in 1537, the loxodroma.

Christopher Rudolff (1500–1545) has introduced the symbol of the second degree radical.

Robert Recorde (1510–1558) has introduced, in 1557, the sign of equality.

Christopher Clavius (1537–1612) has introduced, in 1593, the decimal point.

François Viète (1540–1603) has elaborated the generalized theory of Pitagora and has introduced, in algebra, the trigonometric functions, in 1579.

Ubaldo del Monte (1545–1607), the founder of the theoretical perspective, has published, in 1600, “The treatise Perspectivae libri sex” (Six books on perspective), in which he put the theoretical bases of perspective.

John Nepper (1550–1617) has discovered, in 1614, the logarithms.

Thomas Harriot (1560–1621) used for the first time the symbols "<", ">" in "Artis Analyticae Praxis ad Aequationes Algebraicas Resolvendas", posthumous published in 1631.

Thoma Finck (1561–1656) introduced in 1583, the terms tangent and secant.

Johannes Kepler (1571–1630) introduced in 1604, the term of focus of an ellipse.

William Oughtred (1574–1660) introduced in 1631, the sign of multiplication (×), the sign (±) and in 1657 the sign of division (:). At the same time, he introduced the symbol for parallelism in a posthumous published paper in 1677. John Kersey (1616–1677) has taken the symbol, publishing it in 1673, before W. Oughtred.

Pierre Hérigone (1580–1643) introduced in 1634 the shortening sin.

Günter Edmund (1581–1626) introduced in 1620, the names of cosines and of cotangent. The notation cos belonged to Stephenson (XVIIth century) in 1674.

Etienne Pascal (1588–1651) discovered the snail of Pascal.

Gérard Desargues (1593–1662) published in 1636, the fundamental theorem of the homological triangles. G. Desargues put the bases of the projective geometry of the conics in his paper "Brouillon project d'une atteinte aux événements des rencontres du cone avec un plan" published in 1639 and introduced the notions of: involution, fascicule of lines and fascicule of planes (in 1639).

Renè Descartes (Cartesius) (1596–1650), the founder of the analytical geometry, has discovered, in 1620, the general relation between the number of picks v, of edges m and of faces f of a convex polyhedra and introduced, in 1637, the notion of real number. At the same time, in 1638, he introduced the folium of Descartes, the logarithmical spiral and the system of Cartesian coordinates.

Pierre Fermat (1601–1665) introduced, in 1630, the names of: point of extreme and point of inflexion.

Gilles Roberval (1602–1675) discovered the quadratriques in 1645.

Evangelista Torricelli (1608–1647) introduced: the strophoid, the elongate and shortened cycloids, in 1644.

John Wallis (1616–1703) determined the general equation of the conics and introduced: the negative coordinates and the cubic parabola (in 1656).

Vicenzio Viviani (1622–1703) discovered, in 1692, the curves bearing his name.

Johann Heinrich Rahn (1622–1676) introduced, in 1659, the symbol asterisk (*).

René de Sluse (1622–1685) determined, in 1659, the graphical representation of a given function and defined a family of named the pearls of Sluse by Blaise Pascal (1623–1662).

Blaise Pascal (1623–1662), one of the creators of the integral calculus, discovered, in 1639, the theory regarding the hexagon which is inscribe into a conic (the theory bears his name).

Domenico Cassini (1625–1712) discovered, in 1680, the ovals of Cassini.

Christiaan Huygens (1629–1695) introduced the notions of: evolutes, evolvent and tractors.

Isaac Barrow (1630–1671) introduced in 1670, the notion of axis of coordinate.

Philippe Lahire (1640–1718) treated systematically the epicycloids and the hypocycloids in 1666 and introduced, in 1679, the names of origin and elevation. P. Lahire initiated in 1679, the analytical geometry of the space, determined the parametrical equations of the circle evolvent, in 1706 and introduced the orthoptic centre of a conic.

Isaac Newton (1643–1707) and Gottfried Wilhelm Leibniz (1646–1716) put the bases of the modern infinitesimal analysis. Newton introduced, in 1671, the notion of centre of curvature in a point to a plane curve and determined the expression of the curvature radius.

Gottfried Wilhelm Leibniz (1646–1716) proposed the sign of multiplication and introduced the notions of: parallel curves (in 1692), astroid (in 1715), as well as the names of: abscise, ordinate and coordinates.

Walther Tschirnhaus (1651–1708) elaborated, in 1682, the theory of the caustics.

Jacques I. Bernoulli (1654–1705) introduced the notions of: osculatory circle of a curve (in 1695), trajectory of the angle a (in 1697), probability (in 1705) and integral. At the same time, he discovered the curve named the lemniscates of Bernoulli and the clotoide, and in 1698 he determined the geodesical lines of a cone.

Pierre Varignon (1654–1722) introduced, in 1704, the hyperbolic spiral.

Edmond Halley (1656–1742) proved in 1686, that the stereographical projection of the loxodroma is a logarithmical spiral.

François l'Hospital (1661–1704) determined the formula of the radius of curvature in polar coordinates.

Jean Bernoulli (1667–1748) introduced: the notion of point of return (in 1691), the curve named brahistocrona (in 1696), the concept of function (in 1708) and the notation of the elevation with z (in 1715).

Guido Grandi (1671–1742) introduced the conic helix and the exponential curve (in 1701), studied the rosettes (in 1713), and introduced the clelia, that are rosettes on the sphere in 1728.

William Jones (1675–1749), in the paper "Synopsis palmariorum Matheseus" published in 1706, introduced the symbol pi, this one deriving from the initial of the word peripheries.

Roger Cotes (1682–1716) introduced, in 1714, the curve lituus.

François Frézier (1682–1773) published in Strasbourg, in 1737, the book on stereotomy: "La théorie et la pratique de la coupe des pierres et des bois" and studied the quartics, by means of projection on two plans.

Brook Taylor (1685–1731), introduced the term of escape point.

Colin Maclaurin (1689–1746) introduced the notions of: polar of a point as referred to a plan curve (in 1718), the podara (in 1718), the antipodara (in 1720) and the sinuspiral (in 1742).

Vincenzo Riccati (1707–1775) introduced, in 1757, the hyperbolic functions: sinus and hyperbolic cosines.

Leonard Euler (1707–1783) classified the cubics and the quartics, determined the osculatory parabola of a given curve, introduced the notion of graf (in 1736), gave the formulas of the affine transformation of the coordinates in the plan (in 1748) and the notations ctg and cosec (in 1748). L. Euler introduced the Euler angles and the symbol e, expressed the radius of curvature of the normal section (in 1760), determined the directions of the quadric axes (in 1765), introduced the notion of unfolded surface (in 1771), studied the channel surfaces (in 1776) and the areola representation (in 1777), gave the differential equation of the geodesic lines (in 1779), determined the tangent, the principal normal and binormal (in 1782).

Leonard Euler, the most productive mathematician of the world, published 1200 papers with fundamental results, being followed by Arthur Cayley (966 papers), Louis Cauchy (789 papers) and Henri Poincaré.

Alexis Clairaut (1713–1765) was the founder of the differential geometry, by his book "Recherches sur les courbes à double courbure" published in Paris in 1731, as well as of the spherical trigonometry, in 1733. He determined, in 1739, the geodesics of the ellipsoid and introduced the Clairaut theory in 1733.

Agnesi Maria (1718–1799) introduced, in 1748, the versier conical curve or the loop of Agnesi.

Abraham Kästner (1719–1800) determined, in 1771, the equations of the circular elipse.

Heinrich Lambert (1728–1777) proved, in 1767, the irrationality of the number pi.

Gregorio Fontana (1735–1803) introduced, in 1775, the polar coordinates.

Joseph Lagrange (1736–1813) introduced, in 1773, the polar coordinates in space and determined: the inscribed and circumscribed spheres to the tetrahedron (in 1773), the formula of the distance from a point to a plan (in 1773) and the equation with partial derivates of a minimal surface (in 1774).

Wessel Karl (1745–1818) introduced, in 1797, the notion of versor.

The French scientist Gaspard Monge (1746–1818), the founder of the Descriptive Geometry, introduced, for the first time, the method of the double orthogonal projection, in his lecture Lessons of descriptive geometry, that took place at the Normal Superior School and the Polytechnic School in Paris, in 1795, putting the scientific bases of the plane graphical representation of the objects in the space, based on the method of the double orthogonal projection. Although G. Monge discovered the method of the double orthogonal projection and the principles of the descriptive geometry, after having solved some problems concerning the military fortifications, for 15 years it was forbidden for him to present it publically, as it was considered a military secret. In 1799, he published, in Paris, the book "Géométrie descriptive". G. Monge proved in his descriptive geometry, by using the method of projections as a way to establish

the correspondence between two spaces, that by the orthogonal projection of the points in the space on two perpendicular plans that superpose by a rotation around their line of intersection, the objects in the three-dimensional space can be represented graphically on a plan and any geometrical operations can be performed on them.

At the same time, G. Monge determined: the polar axis (in 1771), the formula of the distance between two points in the space (in 1771), the equation with partial derivates of a unfolded surface (in 1775), the lines of curvature (in 1780), the differential equation of the curvature lines (in 1794), the circular points of a surface, the equations of the line in the space (in 1795), the method of the characteristics (in 1795), the general equation of a cylinder and of a cone. G. Monge extended: the theory of the polar as rapport to a quadric and the theory of d'Alembert. G. Monge introduced: the principal radiuses of curvature (in 1794), the evolutes of a surface and the Monge point of the tetrahedron.

Simon Laplace (1749–1827) introduced, in 1798, the omofocal quadrics.

Simon l'Huilier (1750–1840) introduced, in 1788, the symbol lim for the limit of a function.

Adrien Legendre (1752–1833) introduced the notion of common perpendicular of two lines in the space and discovered, in 1787, in the spherical trigonometry, the theory bearing his name.

Lazare Carnot (1753–1823) introduced the notion of complete and extended quadrilateral, in 1803, the theory Menelau for the plan polygons, in space, and for the conics.

Jean Meusnier (1754–1793) introduced, in 1776, the notion of minimal surface and showed that the helicoid and the catenoid are minimal surfaces.

Professor William Farish (1759–1837), from the University from Cambridge, in the paper "On Isometrical Perspective" (1820), put the bases of the axonometry, without demonstrating the rules that he uses.

Sophie Germain (1776–1831) introduced, in 1831, the notion of median curvature.

Carl Gauss (1777–1855) published, in 1828, the paper "Disquisitiones generales circa superficies curvas", in which he put the bases of a new theory of the surfaces. C. Gauss represented the surfaces in curvilinear coordinates, introduced the total curvature, the curvature of a surface, the spherical function (in 1828), as well as the first and the second fundamental form in the theory of the surfaces.

Olry Terquem (1782–1862) introduced (1848) the name of podaric curve.

Charle Dupin (1784–1873) introduced, in 1813, the notions: focal conics of a quadric, conjugated directions on a surface indicator of a point of the surface, and in 1822, cyclid surfaces.

Jean Victor Poncelet (1788–1867) in the paper "Traité des propriétés projectives des figures" published in Paris, in 1822, put the bases of the projective geometry. He introduced the notions: cyclic points of a plan, limit points of a bundle of circles and the homology.

Cauchy A. Louis (1789–1857) introduced, in 1837, the notion of singular point.

August F. Moebius (1790–1868) defined the general notion of projectivity, introduced in geometry the elements from the infinite and the baricentric coordinates (1827), proved the null correlation (in 1828) and initiated the circular geometry (1846).

Nicolai I. Lobacevski (1793–1856) and János Bolyai (1802–1860) put the bases of a new non-Euclidean geometry, named hyperbolic geometry, in 1829, and in 1831, respectively, independently.

Michel Chasles (1793–1880) defined the omography (1829), discovered the two double points of perspective (1837), introduced the notions of characteristics of a family of mobile curves (in 1864), and in the paper Treatise of superior geometry he dealed with the projective study of the conics.

Pierre G. Dandelin (1794–1847) proved, in 1822, the theory bering his name.

Gabriel Lamé (1795–1870) introduced: the notion of curve bundles (in 1816), net of curves, elliptic coordinates, curves and Lamé surfaces (1818).

Jacob Steiner (1796–1863) introduced the Steiner ellipses (1844), the roman surface (1844), the steinerian curve (1848) and the Steiner point of a triangle (1866). J. Steiner fundamented with Michel Chasles, in 1832, the theory of omography.

Adhémar Barré-Venant (1797–1886) introduced, in 1834, the osculatoric sphere of a curve.

Julius Plücker (1801–1868) introduced: the homogenous coordinates (1829), the notion of focal points of a plan algebraic curve (1834), the cylindroids (1865) and the Plucker relations.

Bellavitis Giusto (1803–1880) introduced: the notion of equipollence (1835) and the complex numbers in geometry (1852).

Hamilton W. Rowan (1805–1865) introduced, in 1846, the notions scalar and vector.

Grassmann H. Günther (1809–1877) introduced, in 1844, the notions of Euclidian space and vectorial space.

James J. Sylvester (1814–1897) introduced, in 1863, the notions: hyperplane and hyperpyramid.

George Boole (1815–1864) created the Boolean algebra and founded the modern mathematical logic.

Ossian Pierre Bonnet (1819–1892) introduced, in 1857, the notion of integral conoid.

Arthur Cayley (1821–1895) introduced in 1843 the term of n dimensions, in 1844 the notion of matrix, elaborated the first projective definition of the distance between 2 points and founded the theory of the elliptical functions in 1845.

Riemann Georg (1826–1866) elaborated, in 1854, a new non-Euclidean geometry, named elliptical geometry. At the same time, he introduced the concept of

Riemann space (1854), used by Albert Einstein (1879–1955) in 1916, to elaborate the theory of the generalized relativity.

William Henry Besant (1828–1917) introduced, in 1869, the name of orthocenter of the triangle.

Eugenio Beltrami (1835–1900) introduced: the pseudo sphere (1868) and the differential operators on a surface (1864).

Richard Anthony Proctor (1837–1888) introduced, in 1878, the curve named nefroid.

Felix Klein (1849–1925) initiated, in 1872, the Program in Erlangen: a geometry is the study of the invariances of a group of transformations, the Euclidian geometry corresponding to the resemblances group and the non-Euclidean geometry, to the automorfic projective transformations (a subgroup of the projective one. F. Klein classified, in 1871, the non-Euclidean geometries in elliptical and hyperbolic, the parabolic geometry being the Euclidian one (P. Ion, 1996).

Ferdinand Lindemann (1852–1939) demonstrated, in 1882, that pi is a transcendent number.

Henri Poincaré (1854–1912) was the founder of the combinatory topology, he created the general theory of the infinite determinants (1882) and introduced the automorphic functions in 1882.

Philbert Maurice d'Ocagne (1862–1938) introduced, in 1883, the notion of simedian.

David Hilbert (1862–1943) elaborated, in 1899, the paper "The fundaments of geometry" that built a new geometry, based on logic and completely eliminating the intuition. Also, he introduced the notion of congruence in 1899, and of Hilbert space, in 1913.

Hermann Minkowski (1864–1909) elaborated, in 1896, the geometry of numbers and established the quadridimensional formalism (Minkowski spaces with four dimensions) – concept used by Albert Einstein to elaborate the theory of the restraint relativity.

René Fréchet (1878–1973) introduced, in 1906, the concepts: metric space, filter and Fréchet space.

Ştefan Banach (1892–1945) introduced, in 1922, the concept of Banach space.

Isaac Jacob Schoenberg (1903–1990) introduced, in 1946, the spline curves.

Benoit B. Mandelbrot (1924–) introduced, in 1975, the fractal notion.

The fundamental theoretical researches in the XXth and XXIst led to: the elaboration of new shapes and geometrical structures, discoveries of geometrical operations, properties and transformations, as well as their generalizations. By methods (descriptive, pure and synthetic, analytic), by types of properties (metric, affine, projective, topological), by number of dimensions (hyperspaces), by types of spatial structures (Euclidian, non-Euclidean, topological, abstract etc), theoretical and subsequent applicative developments have been made possible, to the geometry of the space-time-material of Albert Einstein.

3. SHORT HISTORY OF THE DESCRIPTIVE GEOMETRY IN ROMANIA

In our country, the elements of Descriptive Geometry have started to be thought in Iaşi, at the School for engineers, in 1814, by Gheorghe Asachi (1788–1869) and in Bucharest, by Gheorghe Lazăr (1779–1823) at the Sf. Sava National Engineering School, in 1818 (P. Ştefan, 1982).

The first course on Geometry in Romanian language is "Elements of Geometry", after Legendre, that has been published in 1837 in Bucharest, at the typing house belonging to Eliade, translated by Petrache Poenaru (1799–1875).
Alexandru Orăscu (1817–1894) has translated from French a Descriptive Geometry, after the book of Lefébure de Fourcy, entitled "Tratation on descriptive geometry", printed in 1851, with cyrillic letters.

Gheorghe Ţiţeica (1873–1939), the greatest Romanian geometer, has discovered, in 1911, in the Projective Geometry, the surfaces, the curves and the networks named after him.

In our country, are remarked by studies and original contributions in the Descriptive Geometry: Traian Lalescu, Theodor Angheluţă, Alexandru Myller, Niculae Abramescu, Octav Mayer, Alexandru Pantazi, Gheorghe Vrânceanu, Dan Barbilian, Tiberiu Mihăilescu, Ernest Abason, Gheorghe Nichifor, Adrian Gheorghiu, Virgil Dragomir, Mihail Şt. Botez, Aurelian Tănăsescu, Theodor Nţulescu, etc.

REFERENCES

1. Andonie Şt. George, *Istoria matematicii în România*, vol. I, II, III, Editura Ştiinţifică, Bucureşti, 1965.
2. Both Nicolae, *Istoria matematicii*, Atelierul de multiplicare al Universităţii Babeş-Bolyai, Cluj-Napoca, 1981.
3. Drîmba Ovidiu, *Istoria culturii şi civilizaţiei*, Ediţie definitivă, vol. I..... X, Editurile Saeculum I.O. şi Vestala, Bucureşti, 1997.
4. Pascu Ştefan ş.a., *Istoria gândirii şi creaţiei ştiinţifice şi tehnice româneşti*, Editura Academiei R.S.R., Bucureşti, 1982.
5. Purcaru Ion, Bâscă Octavian, *Oameni, idei, fapte din istoria matematicii*, Editura Economică, Bucureşti, 1996.
6. Taton René ş.a., *Istoria generală a ştiinţei*, vol. I şi II, Editura Ştiinţifică, Bucureşti, 1970.