Table 1: Corresponding average gradients and weights for samples with zero weight.

Case	Weighted average g_m	Weight w_m
$w_\mathrm{a} = 0$	$\dfrac{w_\circ g_\circ + w_+ g_+}{w_\circ + w_+}$	$w_\circ + w_+$
$w_\mathrm{b} = 0$	g_+	w_+
$w_\mathrm{a} = w_\mathrm{b} = 0$	g_+	w_+
$w_\mathrm{a} = w_\mathrm{c} = 0$	$-$	0
$w_\mathrm{a} = w_\mathrm{d} = 0$	$g_\circ$	$w_\circ$

Table 2: Formulas for the weight w'_{01} if the new edge $e'_{01} = (v_0, v_1)$ created by the star-cycle swap, for each degree k. The same formulas are valid for each other edge $e'_{i,i+1}$, except by the indices which are increased by i module k.

k	w'_{01}
2	$w_0 w_1 / w_t$
3	$0.5(w_0 w_1 + w_1 w_2)/w_t$
4	$(w_0 w_1 + 0.5(w_0 w_2 + w_1 w_3))/w_t$
5	$(w_0 w_1 + 1.1690(w_2 w_4 + w_0 w_2 + w_1 w_4))/w_t$
6	$(w_0 w_1 + 2 w_5 w_2 + 1.5(w_5 w_1 + w_0 w_2))/w_t$

Table 3: Tested integration methods.

Tag	Description	Type
AT	Diffusive Affine Transform [1, 2]	Poisson - direct solver
AM	M-Estimators [1, 2]	Poisson - direct solver
DT	Isotropic Total Variation [3]	Total Variation (iterative)
DL	L^1 Functional [3]	Iterative split-Bregman solver
MS	Weighted multi-scale on grid [4]	Poisson grid multi-scale
MG	Weighted multi-scale on mesh	Poisson mesh multi-scale

Table 4: Absolute and relative root-mean-square errors of each method for the test datasets, without noise.

	`spdome`		`mixwav`		`cbabel`	
Method	e	e/R	e	e/R	e	e/R
AT	1.82	5.2%	0.89	2.3%	0.02	0.1%
AM	0.58	1.6%	0.46	1.2%	0.02	0.1%
DT	0.05	0.2%	0.02	0.0%	4.51	18.55%
DL	0.04	0.1%	0.67	0.0%	19.90	102.2%
MS	0.19	0.5%	0.36	0.9%	25.31	134.8%
MG	0.04	0.1%	0.02	0.0%	0.03	0.0%
	`cpiece`		`bebust`		`dtbust`	
AT	0.15	0.3%	1.59	11.07%	0.64	2.5%
AM	0.15	0.3%	0.30	2.0%	0.71	2.8%
DT	0.89	17.8%	1.62	10.9%	0.46	1.8%
DL	4.32	104.3%	1.28	8.8%	5.46	23.6%
MS	5.26	138.4%	1.02	6.4%	2.99	12.4%
MG	0.00	0.0%	0.87	5.4%	0.39	1.5%

Table 5: Absolute and relative root-mean-square errors of each method for the test datasets, with 30% of Gaussian noise added.

	`spdome`		`mixwav`		`cbabel`	
Method	e	e/R	e	e/R	e	e/R
AT	3.30	9.8%	4.75	13.0%	0.80	3.0%
AM	0.64	1.8%	0.51	1.3%	0.86	3.3%
DT	0.46	1.3%	0.37	0.9%	12.47	58.6%
DL	0.48	1.4%	0.92	2.4%	24.36	129.7%
MS	0.34	0.9%	0.44	1.1%	25.36	135.1%
MG	0.39	1.1%	0.34	0.9%	0.76	2.9%
	`cpiece`		`bebust`		`dtbust`	
AT	0.55	10.0%	1.94	13.9%	1.22	4.9%
AM	0.54	9.9%	0.40	2.7%	0.71	2.8%
DT	1.46	30.2%	1.21	8.2%	1.21	4.9%
DL	4.46	114.3%	2.26	15.5%	9.86	45.1%
MS	5.25	137.9%	0.90	5.6%	2.98	12.3%
MG	0.46	8.7%	0.93	5.8%	0.59	2.3%

Table 6: Time costs for the six methods on the `spdome` and `dtbust` datasets.

spdome						
	time					
N	AT	AM	DT	DL	MS	MG
$64 \times 64 = 4096$	0.04	0.04	0.05	0.07	0.02	0.02
$90 \times 90 = 8100$	0.07	0.09	0.11	0.11	0.04	0.04
$128 \times 128 = 16384$	0.17	0.15	0.22	0.19	0.09	0.10
$256 \times 256 = 65536$	0.95	0.73	0.51	0.81	0.42	0.51
$360 \times 360 = 129600$	2.50	1.77	1.44	1.66	0.82	0.99
$512 \times 512 = 262144$	6.28	4.41	3.13	5.34	1.70	2.07
$1024 \times 1024 = 1048576$	45.36	29.24	16.98	22.05	5.22	8.43
$1280 \times 1280 = 1638400$	78.89	50.82	31.63	35.08	8.96	11.86
$1536 \times 1536 = 2359296$	144.97	85.11	77.19	52.18	9.74	17.30
$1792 \times 1792 = 3211264$	222.17	126.49	72.89	69.73	13.25	23.90
$2048 \times 2048 = 4194304$	351.04	194.06	119.93	95.32	21.92	30.88
dtbust						
	time					
N	AT	AM	DT	DL	MS	MG
$64 \times 64 = 4096$	0.04	0.06	0.04	0.07	0.02	0.01
$90 \times 90 = 8100$	0.05	0.07	0.06	0.09	0.04	0.03
$128 \times 128 = 16384$	0.09	0.11	0.09	0.18	0.08	0.08
$256 \times 256 = 65536$	0.48	0.44	0.34	0.69	0.38	0.38
$360 \times 360 = 129600$	1.03	1.01	0.74	1.33	0.79	0.80
$512 \times 512 = 262144$	2.17	2.24	1.79	4.70	1.53	1.67
$1024 \times 1024 = 1048576$	13.16	13.80	10.26	19.83	5.27	6.40
$1280 \times 1280 = 1638400$	23.41	22.59	19.80	32.82	8.12	9.80
$1536 \times 1536 = 2359296$	40.98	38.34	33.12	47.31	9.19	14.38
$1792 \times 1792 = 3211264$	59.17	57.98	44.50	59.93	16.18	19.12
$2048 \times 2048 = 4194304$	92.51	89.46	72.78	79.54	21.10	25.03

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